# Monitoring Sea Level Using GPS – The Difference between the Mediterranean and the Red Sea Levels as a Test Case

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**Key words:** Sea level, tide gauge, GPS

## SUMMARY

The maritime distance between Tel-Aviv and Eilat is thousands of kilometers, while the ground distance is about 400km. In such a case, the height difference between the two seas can be obtained from tide gauges situated along the shores and precise leveling between the tide gauge benchmarks. This approach besides being very expensive and notoriously time consuming is also expected to deliver height difference accuracy level of decimeter level for such a distance.

GPS is a tool that can easily and quickly solves vectors longer then 400km with centimeter level of accuracy and can measure point height changes relative to the reference ellipsoid WGS84. Although in the past two decades GPS penetrated massively almost every field of geodetic measurements, it is still almost not in use in the field of sea level assessment. Attempts of using GPS equipped buoys for the determination of precise sea level (at centimeter level) were successful and suggest that if carefully used, GPS is capable of replacing the conventional tide gauges.

The paper describes the development of a GPS based tide gauge (GPTG) prototype and its successful operation. Two field tests were carried out, the first one was a proof of concept, and demonstrated measurement ability with the accuracy of close to 1cm. The second test was carried out with a goal for determination the capacity of the GPTG to connect two distinct tide gauges, and its ability to measure accurately the level difference between the two distinct bodies of water - the Red Sea and the Mediterranean Sea. Examining the results of the prototype test and the second test resulted in a conclusion that the use of a GPS based Tide Gauge system for the determination of sea level changes is possible, and that its accuracy level (averaged) is equal to a float based tide gauge. More than that, an absolute change of sea level should be easier to be determined by the GPTG.

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## 1. INTRODCTION

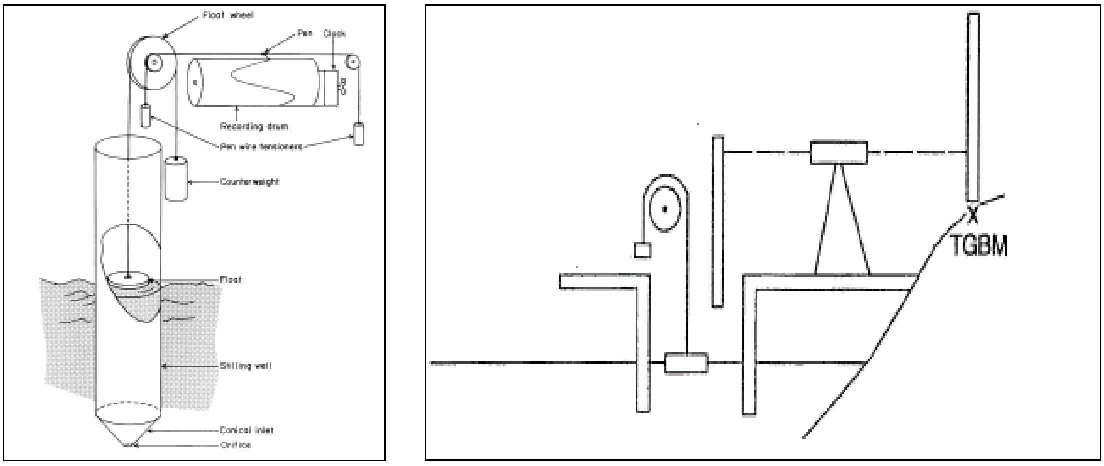
A geodetic datum is a set of parameters and control points used to define the size and shape of the earth and the origin and orientation of the coordinate systems used to map the earth. The datum is the basis for a coordinate system. Traditionally, geodetic networks are divided in two, horizontal and vertical network. The datum of horizontal network is mainly defined by mathematical manipulations. The Datum definition of vertical network is more complicated than datum definition for horizontal network since "height users" such as engineers, geophysicists, oceanographers and so forth, demand physical meaning for vertical coordinates. The datum, to which those systems based on are referred to the mean sea level.
Nowadays assessment of sea level is done using sea level measurements produced by tide gauges (or mareographs) located in protected points along the coasts. A mareograph
is defined as a mechanism, which can determine the vertical distance (height) between the sea surface and an adjacent benchmark (BM). There are several types of tide gauges for various uses, beginning with "staff tide gauge" - used in conventional hydrographic surveys., and ends with, instruments mounted below sea surface and measure the pressure of the water-column. Generally, modern tide gauges depend on "more sophisticated and less mechanical" methods for measuring and recording sea level, methods like Doppler and recording pressure changes (implies on the height changes of the water column above them).
In the past two decades a new measuring technique was developed – GPS. The Global Positioning System – provides the ability to specify an exact spatial location of a point. The location is referred directly to the WGS84 (World Geodetic System 1984) datum. In the past two decades GPS penetrated massively almost every field of geodetic measurements. However it is still not in use in the field of sea level assessment. Attempts of using GPS equipped buoys for the determination of precise sea level (at the 1-cm level) were successful and suggest that if carefully used, GPS is capable of replacing the conventional tide gauges.
In order to assess the possibility of using GPS measurements as a reliable and easy to use tool, it was decided to merge the GPS measurement technique into a float based tide gauge system.

### 1.1 Tide Gauges - principles of operation

This section will describe the mounting and operating principles of tide gauges. We will refer only to a "float based tide gauge", the less sophisticated, but most common type of instrument. The reason for choosing this kind of TG to be the infrastructure of our research is its simplicity, not only in the mechanical structure but also in the mounting procedure. The basic description of a float based analogue tide gauge is shown in figure 1a.

## 1.2 The Float, the recording drum and the stilling well

A float is located on the surface of the water and connected by a cable and a set of pulleys to a weight, a pen and to a recording drum. As the water rises or descends, the cable movement over the pulley will create an angular movement that is proportional to the change in water level. In analogue devices the curve that will be drawn is a continuous record of water height against time. It should be point out that the short period changes such as oscillations in harbors, and surface waves contaminate the results by adding short period changes to the long term desired data. Therefore, to eliminate this contamination, the float is placed inside a Stilling Well (SW). The SW is a large diameter pipe, which mechanically restricts the flow of water into and out of the well, and this creates an environment that is almost free of disturbances. The ratio between the inlet of the stilling well and its diameter will set the level of compensation. The recommendation in the Intergovernmental Oceanographic Commission (IOC) manual on sea level measurement and interpretation (IOC 1985) is 1:10 but SW must be checked empirically due to local changes. Another component of the tide gauge system is a BM, a well-defined point with a defined height, which provides the reference point for the sea level measurements. The BM is connected to the tide gauge via a Tide Stuff (TS) or directly measured into the tide gauge system. Although the BM is not a physical part of the mechanism, its establishing is crucial for the operation of the TG. Figure 1b demonstrates the tying of a TGBM to the TG




(a) A float based analogue tide gauge description

(b) TGBM tying to TG

**Figure 1**- The operation principles of float based tide gauge (figures adapted from IOC 85).

## 1.3 The use of GPS for sea level assessment

GPS penetrated almost every field of geodesy. Until recent years the technological progress was not accompanied by a parallel philosophical perception as mentioned by Steinberg and Papo (1998). The control networks continued to be divided in two – vertical and horizontal.

This duality could be solved by transferring the traditionally orthometric vertical networks to the ellipsoidal reference system. It is true that many height users prefer the orthometric system, but as for Mitchum (1997), Neilan et al (1997) and others, sea level or sea surface should be connected to the ellipsoidal height system for several reasons such as assessing the absolute change of mean sea level (MSL). Teferle (2003) also vastly describes the idea of combining the GPS into the array of sea level monitoring. Two methods are found to relate sea level with ellipsoidal heights, the more accepted one - is to establish a continuous permanent GPS station (CGPS) near the TG and conduct high precision leveling between the station and the TGBM repeatedly (Bingle, Dodson and Teferle 2001). The secound method is to place GPS buoys and finds the compatibility of their data with the orthodox TG's data (Chang-Fang Lo and Ming Yang 2000). Checking the results of the latter show that high precision measurement (1cm level) could be achieved with complex buoys containing various tilt sensors. This research was aimed at developing, building and operating a low-cost, precise (1cm level) GPS based TG (GPTG).

## 2. PROTOTYPE DEVELOPMENT AND MOUNTING

Although, the GPTG was to be based on a float tide gauge (TG), few adjustments were to be done. The SW was build from a PVC 10" diameter pipe with flat bottomed and side inlets, as shown in figure 2a. After consulting with research division of the Survey of Israel (SOI) the ratio between the inlet and pipe diameters was set to a 1:7, different from the recommendation in the IOC manual on Sea Level Measurement (2002) but empirically tested in SOI TG's to sufficiently suppress the surface disturbances. Data logging is an inherent part of a GPS receiver so this part (the recording drum in the analogue devices) was skipped. The connection between the antenna (and by that the phase center) and the buoy was rigid and transferred the vertical movement from the buoy to the phase center. Another problem occurred with this prototype device is the need to restrain the horizontal movement of the antenna pole, the solution was to place centering disks inside the SW as demonstrated in figure 2b.

(a) – The stilling well with flat bottomed side inlets

(b) – The centering disk

**Figure 2** – The PVC GPTG stilling well.

The total horizontal movement of the pole was less than 5mm, which contributes only 1-2mm to the height's calculation error. The buoy was manufactured from a plastic container filled with polyurethane foam (to add rigidity), the antenna pole was molded into the foam. The pole was a 1.5-4.5m long, the exact height was to be determined at the site, depends on distance from sea surface and disturbances. The GPS receiver was an Ashtech Z-Surveyor and was placed inside a closed wooden box, with geodetic AeroAntenna (AT2775-42).

### 2.1 Installation location choosing

After a thorough check the Tel-Aviv marina was chosen to be the installation location. For the reasons of being close to a reliable functioning TG, SOI is operating a TG there for several years, providing an easy and convenient access and installation, and finally only 3km away from the IGS (International GNSS Service) site TELA. Figure 3 show the GPTG installed near the traditional SOI-TG.

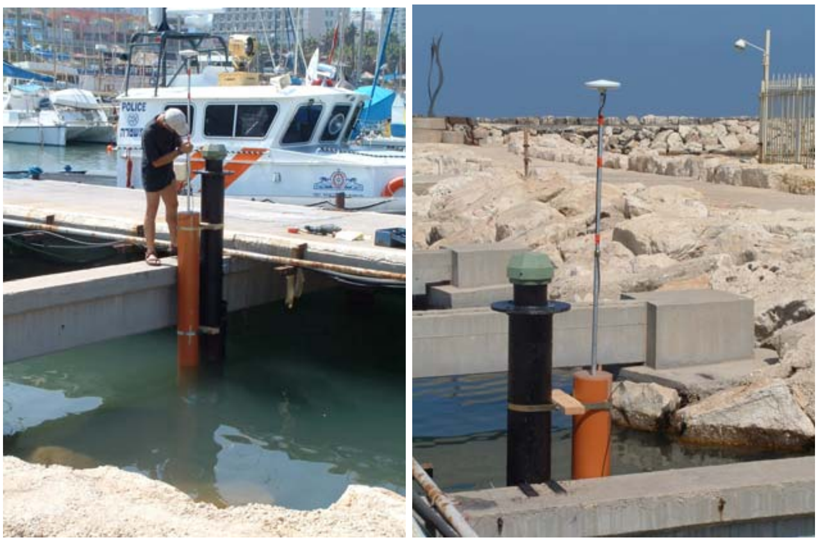


**Figure 3** – GPTG installed near the traditional SOI-TG.

## 3. FIRST FIELD TEST - PROOF OF CONCEPT

The installation of the GPTG prototype took place in the Tel-Aviv marina on July 5$^{th}$ 2004, the test last 29 hours, the epoch interval set to 10 seconds and the cutoff angle to 15 degrees. A total of 10,271 epochs were measured.

### 3.1 Data comparison methodology – GPTG versus TG

The first step is to solve the GPS vectors between TELA and GPTG and attach height to each time epoch. The vectors were post processed by commercial software TTC (Trimble Total

Control). The measured ellipsoid height for every epoch ($h_i$) is approved only if the PDOP is fewer than 6; the number of satellites is equal or great than 4 and the linear height change between two consecutive epochs is less than 8cm. In mathematical form, an ellipsoid height observation is approved if:

$$if\,[(PDOP < 6)and(no\_of\_sat \geq 4)and(\left|h_{i+1} - h_i\right| < 0.08)] \Rightarrow h_{i\_approved}$$

The first field test was a "proof of concept" and didn't make any use of geoid's undulation model. Therefore it seems that no comparison between the heights derived from the GPTG (ellipsoidal) to the heights derived from the TG (orthometric) could be done.

Geoidal undulation (N) is the practical difference between ellipsoidal height (h) and orthometric height (H) and equal to:

$$N = h - H \tag{1}$$

The sea level in epoch i+1 is calculated relative to sea level in epoch i by adding the level change between epoch i to i+1, and it is described in both height systems as

$$\begin{aligned} h_{i+1} &= h_i + \Delta h_{i \to i+1} \\ H_{i+1} &= H_i + \Delta H_{i \to i+1} \end{aligned} \tag{2}$$

Therefore, the geoidal undulation in epoch i+1 is

$$N_{i+1} = h_{i+1} - H_{i+1} = h_i + \Delta h_{i \to i+1} - H_i - \Delta H_{i \to i+1} \tag{3}$$

If we assume that the GPTG and the TG measure the same change in sea level differs only by a random error $\varepsilon$ then:

$$\Delta h_{i \to i+1} = \Delta H_{i \to i+1} + \varepsilon \tag{4}$$

The geoidal undulation in epoch i+1 is equal to:

$$N_{i+1} = N_i + \varepsilon \tag{5}$$

Since $\varepsilon$ is normal distribution with $\varepsilon \sim N(0, \sigma^2)$ then $E(N) = N$. That's mean that for a specific point, only a constant will separate the TG and the GPTG results, in our case this constant contains the antenna height (which we didn't measure) and the undulation at the location.

Atmospheric parameters such as wind and barometric pressure shouldn't be considered for they are the same for both instruments. In order to compare the sea level measured by the TG and the GPTG we should bring them to common denominator in time and height scales. The SOI-TG programmed to measure in interval of 1 minute and recording means ($\overline{H}_t$) of 5 minutes as

$$\overline{H}_t = \frac{\sum_{i=t-2.5}^{t+2.5} H_i}{5} \tag{6}$$

where t is in minutes.

The GPTG measurement and recording interval is 10 seconds so we had to thin out the GPS data set in order to compare it with the TG data set.

Therefore, The mean of ellipsoid height ($\overline{h}_t$) is given by

$$\overline{h}_t = \frac{\sum_{i=t-2.5}^{t+2.5} h_{i\_approved}}{n_{approved}} \quad (7)$$

The final time synchronization check was made through a sampled cross correlation factors (XCF) calculation, first the sample cross-covariance was calculated

$$c_{xy}(k) = \begin{cases} \sum_{t=1}^{n-k} \left(h_t - \overline{h}\right)\left(H_{t+k} - \overline{H}\right)/n....[k = 0,1,2.........(n-1)] \\ \sum_{t=1-k}^{n} \left(h_t - \overline{h}\right)\left(H_{t+k} - \overline{H}\right)/n....[k = -1,-2...,-(n-1)] \end{cases} \quad (8)$$

and when "Lag 0" stands for k=0 the XCF itself for "Lag 0" is:

$$r_{hH}(0) = c_{hH}(0)/\sqrt{c_{hh}(0)c_{HH}(0)} \quad (9)$$

The term "Lag" stands for time offset between data sets. We can use (9) with "Lag 1" - one time interval (in our case 5 minutes) difference or with any other "k". With "Lag 0" there is no offset in time between the two data sets.

Concerning the height scale, a mean of the geoidal undulation ($\overline{N}$) was calculated based on the difference of a pair of variables $(h_i, H_i)$ and reduced from each ellipsoidal height observation

$$h_i' = h_i - \overline{N} \quad (10)$$

The normalized data ($h_i'$) is similar to transforming ellipsoidal heights to orthometric heights. The result was a normalized set of GPS data, ready to be compared both in time and in scale.

### 3.2 Results of first field test

The time synchronization check showed an almost perfect match (0.997) in "Lag 0" – means that both data sets are almost 100% synchronized. Figure 4 shows the SOI-TG and GPTG data; one should notice the scale and the fact that even by visual check, the results shows almost a perfect match. Figure 5 shows the difference between the data sets which is close to 1cm (mean).

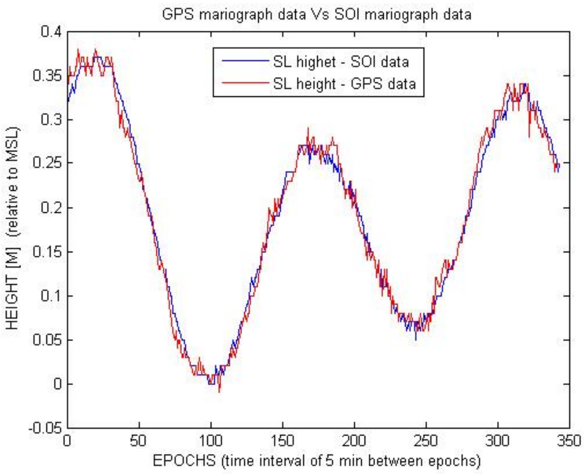



**Figure 4** – Comparison of SOI-TG data versus the GPS mariograph data.

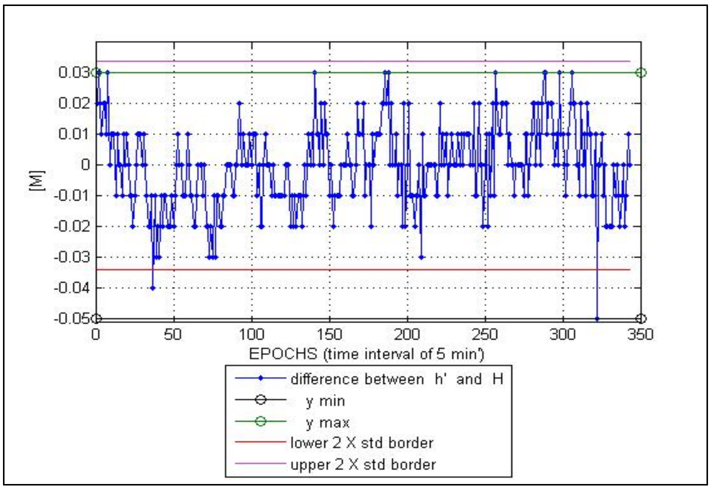



**Figure 5** – Difference between normalized GPS measurements and SOI.

We shall now examine the difference between the data sets by statistical tests of hypotheses. When $\mu_1 = E(h')$ and $\mu = E(H)$ the null hypothesis

$$H_0: \quad \mu_1 - \mu = 0$$

can be tested against the alternative hypothesis

$$H_1: \quad \mu_1 - \mu \neq 0 .$$

If the null hypothesis is accepted there is no difference between data sets. Let $D_i$ is the difference between the normalized ellipsoidal height and orthometric height, $D_i = h'_i - H_i$ and $\bar{D}$ is the mean of the all $D_i$. The test statistic with n-1 degrees of freedom is:

$$t = \frac{\bar{D}\sqrt{n}}{\sqrt{\frac{\sum_{i=1}^{n}\left(D_i - \bar{D}\right)^2}{n-1}}} \sim T_{\alpha,n-1} \qquad (11)$$

If $t > T_{\alpha,n-1}$, the null hypothesis is rejected with probability $\alpha$. With $\alpha = 5\%$ we get $T = 1.645$ and $t = -0.3209$ mean that the null hypothesis is accepted. The results from both instruments can be treated as identical with standard deviation of 13mm and correlation factor $\rho$ of 0.993. When coming to check the results one should also pay attention to the fact that the measurement resolution is about 1cm for the TG and the GPTG.

## 4. IMPLEMENTATION OF THE GPTG FOR SEA LEVEL DIFFERENCE MEASUREMENT

After the first field test proved that a GPTG could provide sea level measurements at the same accuracy level as a conventional TG, an implementation of the device for connecting two distinct tide gauges was carried out. Its goal was determination of the GPTG ability to measure accurately the sea level difference between the Red Sea and the Mediterranean Sea. A second device was built and installed at the Inter University Institute (IUI) in Eilat. Same principles guided us in choosing this location but it differed from the Tel-Aviv marina by its fewer defenses for the GPTG, as shown in figure 6 – the location is at an end of a 25m peer built from the IUI shore. The GPTF in Eilat was situated less than 2 km from an IGS station ELAT.

**Figure 6** - Inter University Institute, the installation place of the Read Sea GPTG.

The test took place between the 26/09/04 and 28/09/04. The instruments were operated in Tel-Aviv for 52 hours and in Eilat for 39 hours. The distance between the two locations approximates 350km, but the naval distance is thousands of kilometers. We applied the same configuration for the receivers, both of Ashtech Z-Surveyors with geodetic AeroAntenna (AT2775-42). Figure 7 shows the sea level variations of the Red Sea and the Mediterranean Sea in ellipsoidal reference system.

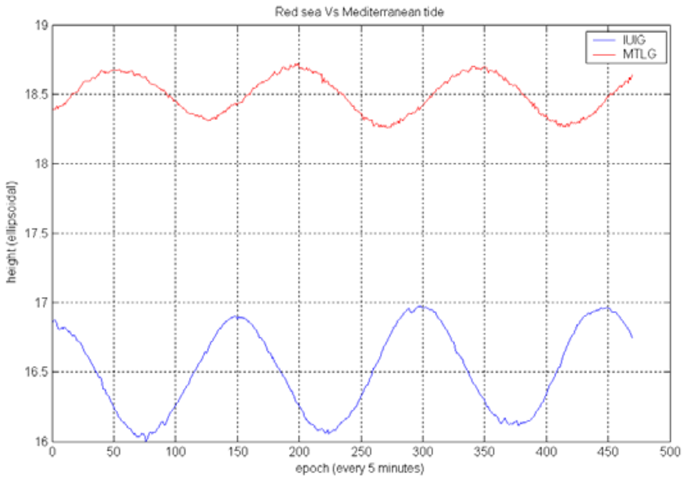



**Figure 7** – Sea level variations of the Mediterranean Sea (red line) and the Red Sea (blue line) in ellipsoidal reference system.

### 4.1 Two distinct bodies of water – water level comparison

The raw GPS data were checked for multipath, which is believed to be of a great interference in marine environment. The TEQC software (http://www.unavco.org) developed by UNAVCO was used to check the data for multipath and discovered no unusual effect. A GPS vector was calculated between TELA and ELAT, by using SOPAC (http://sopac.ucsd.edu/) adjusted coordinates. Then each GPTG was solved against its neighboring station. This calculation enabled to connect the two GPTG's. Sea level calculations were made using a mean sea level for the measured period.

### 4.2 External influence

Winds affect the change in sea level through generating waves and piling the water, and are very much direction dependant, therefore their effect is difficult to model and wasn't dealt with. A point to emphasize is while conducting the field tests, no unusual atmospheric conditions (storms or high water) were pointed out. The barometric pressure, on the other hand, was taken into consideration through the "inverse barometric pressure formula" (Pugh, 1987):

$$\Delta h = -\Delta Pa / \rho \cdot g \tag{12}$$

With sea water density of $\rho = 1.0265_{g/cm^3}$ and $g = 9.80_{m/sec^2}$ we will get

$$\Delta h = -0.995 \Delta Pa \tag{13}$$

$-\Delta Pa$ is the change in millibars relative to standard atmospheric pressure of 1013 millibars an $\Delta h$ is the height change in centimeters. The thumb rule is that for a 1-millibar increase in pressure, the sea surface will decrease 1cm. This is of special importance when comparing two distinct points as in the second field test.

To compare the Red Sea level and the Mediterranean Sea we used undulation values derived from a recent SOI geoid model (Tuchin, 2006). The undulation values and their accuracy can be seen in Table 1.

Table 1 – Coordinates and undulation of the GPTG locations.

| | **Lat** [dd] | **Long** [dd] | **Undulation** [m] |
|---|---|---|---|
| **Tel-Aviv (**Mediterranean Sea) | 32.0871 | 32.7677 | 18.57±0.08 |
| **Eilat** (Red Sea) | 29.5018 | 34.9176 | 16.50±0.04 |

The level difference between the Red Sea and the Mediterranean Sea, after implementing the external influences was found to be $-0.035\text{m}$.

## 5. SUMMARY AND CONCLUSIONS

Developing and manufacturing a prototype of a GPS based, float operating tide gauge was successful. The instruments were located in different conditions (close marina and open sea) and operated successfully in both .The comparisons between the Red Sea level and the Mediterranean Sea level showed a difference within the error margin of the undulation model. Checking the results shows that GPTG seems to have met the performance of the traditional float operated mariographs. It seems that the GPTG is capable of delivering the same level of accuracy (1cm), and as reliable results as its opponent. For an absolute determination of a Mean Sea Level change – the GPTG seems to be more than an easy to operate, reliable and low cost tool even when considering the exiting orthometric data. For a vertical network control – while the main error is still coming from the undulation model the GPTG can supply a rough error control but not accurate enough for geodetic use.

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## BIOGRAPHICAL NOTES

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