PRECISE METHOD GYROTHEODOLITE AZIMUTH DETERMINATION

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Abstract

The Wild GAK1, Gyrotheodolite is a surveying instrument used to orientate an underground survey baseline relative to true North. A new mathematical model concerned with the motion of the moving mark was derived. This model deals with the general and practical cases. For example, when the gyro is used in a tunnel where the battery, which runs down with time, is the main source of power. A new method of time capture and "data" processing is described. This method requires a video camera, video imagery, frame analysis and a computer. The method may be used in many practical applications but it must be acceptable for mine safety for electric equipment if used in a mine. The method leads to a great increase in the quantity and precision of time observations. The observations have a precision five times better than those observed by manual methods. After processing, the time and scale divisions data are used in a rigorous mathematical model and processed by least square techniques. This leads to high quality solutions and statistical assessments. Least squares adjustments showed that the computed values of the midpoint of swing might be determined to standard deviations of less than one second of arc.

1. Description of the observations and experiments:

Two sets of time observations were used in this research. In the first set, the battery was not full charged and after two and half-hours was flat. This is similar to the practical case, which may occur in the field. In the second set, the battery was fully charged at the start. The method used in this work required a video camera, Fig. 1 to observe the oscillations of the moving mark. The camera was set to look through the gyro eyepiece onto the gyro scale and also, to record each time the moving mark crossed a scale division. The gyro spinner was run up to its full speed. The only power source, which drove the motor was a battery. The mast was carefully dropped to make sure that the moving mark did not go beyond the limit of the scale divisions. The moving mark was allowed to settle down for a few minutes before observation were taken. The observations are summarised in the table below.

Table 1 Summary of observations

Set of observations	Range of oscillations	Number of readings	Observations per oscillation	complete oscillations
1	-11 +10	552	44	> 12
2	-11 +11	1165	46	> 25

Table 1 shows that more than twelve oscillations of the moving mark were completed in the first set of observations and more than twenty-five oscillations were completed in the second set of observations. Only one or two oscillations of the moving mark are involved in conventional methods of observation. Therefore, this method leads to a substantial increase in the quantity of observations. In this method, many observations of time may be taken in a single oscillation of the moving mark. The readings of each scale division are repeated many times, therefore, the degrees of freedom are greatly increased.

1.1 Automated Data Capture of the Maximum Available Data:

Observations of time at scale divisions are captured on videotape. The time was recorded when the moving mark crossed a scale division. In the laboratory, Fig. 2 a time code was put on the videotape to extract each time observation. With the aid of video frames, the quality of timing improved from approximately 0.2 seconds, at best by manual methods, Gregerson (1974) to the time equivalent of one frame. One frame = 1/25 seconds = 0.04 seconds. Automated data capture leads to a great increase in the quantity and quality of observations. A new mathematical model eq. (1) was derived. This model takes into account the changes in the physical environment of the suspended gyroscope, that is, to account for accelerations.

Fig. 1 Gyrotheodolite observations with video camera

1.2 Semi-Automated Data Processing:

Video imagery using frame analysis is selected and the time data is extracted with the aid of a microcomputer, “laptop”, see Fig. 2. After processing the time data, it is used in a rigorous mathematical model and processed by least square techniques, which lead to high quality solutions and statistical assessments.

Fig. 2 Laboratory data processing

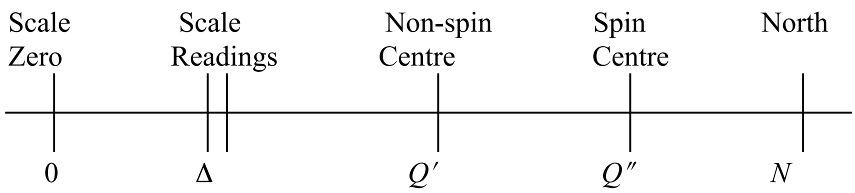
2. Procedure of determining the north::

The theodolite is directed approximately to North to within a few minutes of arc. The gyro spinner is run up to full speed. The motor is driven by a battery, which runs down with time. The mast is dropped carefully, and the gyro is released to seek true North and oscillate about the meridian. In fact, the gyro swings under the influence of many forces. Some of those forces are the precession torque due to a couple applied to the spinner axis, the torque due to the twisting tape, decelerating forces applied to the spinner due to the change in the rate of its angular velocity and finally the gravity forces. Therefore, the swinging gyro defines the direction in which the forces are in equilibrium rather than the direction of the North. The problem of determining azimuth is broken down into two parts, Breach (1983). Firstly, determine the mid-point of swing in the spin and non-spin mode, eq. (1). Secondly, relate the mid-point of swing, Q determined from the last equation to equation of Z, the reading on the horizontal circle of the theodolite, which is equivalent to North. See Fig. 3.

$$\sum[A_i e^{-\lambda_i t} \cos((\theta_i + \theta_i' t)(t - B_i))] + Q - \Delta = 0 \quad i = 1,\ldots..10 \qquad (1)$$

Where:
A_i: Magnitudes of the oscillations.
θ_i: Frequencies of the oscillations.
θ_i': Rates of change of the frequencies of the oscillations due to the change in the angular velocity of the gyroscope spinner.
λ_i: Coefficients of damping forces.
B_i: Times at a positive turning point.
t: Time.
Q: Mid-point of swing.
Δ: Scale readings.

Fig. 3 Diagram of the scale of Wild GAK1 gyroscope



Δ, scale reading, the moving mark appears on the gyro scale as a double line. Q' is the position that the moving mark would take when the gyro is in the non-spin mode. This position would not be at the scale zero because the system cannot be adjusted that precisely. Q'' is the position that the moving mark would take when the gyro is in the spin mode. N is the direction of astronomical North. The position of scale zero depends on determining of two torque affecting on the gyroscope oscillations. The first torque is due to precession, which may be written as according to Thomas (1982):

$$\Omega_1 = P_a \varpi \cos\phi \sin \varepsilon \qquad (2)$$

Where, ϖ the angular velocity of the earth and ϕ astronomical latitude, are constants.
P_a, is the angular momentum of the spinner.
ε is an angle between the spinner axis and the North.
Since ε, in radians is a very small angle, the above equation becomes:

$$\Omega_1 = K_1 \varepsilon \qquad (3)$$

Where:
Ω_1 is a torque due to precession.
K_1 is the precession torque per unit angle e.g. arc-seconds. K_1 is constant at given latitude.
From Fig. 3, eq. (3) may be written as:

$$\Omega_1 = K_1 (N - \Delta)s \qquad (4)$$

Where, s is the value of one scale unit in angular measure, for example, arc seconds.
The second torque is due to tape twisting, which may be written as:

$$\Omega_2 = K_2 \varepsilon' \qquad (5)$$

Where:
Ω_2 is the torque due to tape twisting.
K_2 is the tape twisting torque per unit angle e.g. arc-second.
ε' is the rotation from the tape zero axis.
From fig. 3, eq. (5) may be written as:

$$\Omega_2 = K_2 (Q' - \Delta)s \qquad (6)$$

The total torque is the sum of the two torque Ω_1 and Ω_2:

$$\Omega = \Omega_1 + \Omega_2 = K_1 (N - \Delta)\, s + K_2 (Q' - \Delta)s$$

$$\Omega = s[K_1 N + K_2 Q' - \Delta(K_1 + K_2)]$$

$$\Omega = s(K_1 + K_2)\, [(K_1 N + K_2 Q')/(K_1 + K_2) - \Delta] \qquad (7)$$

The ratio between the two torque Ω_1 and Ω_2 may be written as:

$$K = K_2/K_1 \qquad (8)$$

At the mid-point of swing, the centre of oscillation, the total torque is zero, that is, when $\Delta = Q''$: Substitute Δ in eq. (7) by Q'' and put Ω equal to zero, we get:

$$(K_1 N + K_2 Q')/(K_1 + K_2) - Q'' = 0 \qquad (9)$$

Then:

$$Q'' = (K_1 N + K_2 Q')/(K_1 + K_2) = (N + (K_2/K_1)\, Q')/(1 + K_2/K_1) \qquad (10)$$

Taking account of eq. (8), eq. (10) becomes:

$$Q'' = (N + K\, Q')/(1 + K) \qquad (11)$$

Where, K is the constant of the torque ratio. Eq. (11) may be re-arranged to get:

$$N = Q''(1 + K) - K\, Q' \qquad (12)$$

Suppose Z, is the reading on the horizontal circle of the theodolite, which is equivalent to North. Z may be written as:

$$\mathrm{Z} = \Theta + s\, N + E \qquad (13)$$

Where, Θ is the reading on the horizontal circle when the theodolite is clamped up ready for observations of the gyro. E is an instrument constant. Substitute eq. (12) into eq. (13) to get:

$$\mathrm{Z} = \Theta + s\, Q''(1 + K) - s\, K\, Q' + E \qquad (14)$$

Finally, the azimuth of the line of a reference point may be written as:

$$A_Z = H_R - \Theta - s[Q''(1 + K) - K\, Q'] - E \qquad (15)$$

Where, H_R is the observed horizontal circle reading to a reference point.

3. Practical applications and limitations:

The observational process consists of a pre-orientation towards the North; this can be done from the use of the gyro in the unclamped method. The mast is dropped in the spin mode and the observer tries to rotate the theodolite slowly to keep following the moving mark on the zero division. At the extremity of the oscillation the theodolite is clamped up and the horizontal circle is read. A provisional estimate of North may be achieved by taking the mean of two successive readings. The pre-orientation to North must be determined as accurately as possible to ensure accurate azimuth determination and to reduce errors due to the centrifugal force of earth rotation, Hodges (1996). This process may take about 15 minutes and achieve a precision of a few minutes of arc. The Gyrotheodolite may be set up on a tripod or pillar. A pillar is preferable, because a wooden tripod may twist in damp or sunny conditions and may be knocked easily. The duration of 15-20 minutes of pre-orientation of the North is essential for the instrument to reach its equilibrium temperature before making any observations. Then, the observer is ready to make the required observations to determine the azimuth to a reference point. These observations include:

- Two observations of the horizontal circle readings H_R to a reference point.
- Two observations of the horizontal circle readings Θ_1 and Θ_2 with the theodolite pointing about half a degree to the West and East of North.
- Observations of time versus scale divisions for determining the mid-point of swing twice in the spin mode and once in the non-spin mode. To reduce timing errors, a video camera is used to observe and record time on videotape at each instant the moving mark crosses a scale division. The data is extracted with the aid of a "laptop" computer and by using the video imagery with frame analysis. The video camera is set up carefully to focus on the eyepiece of the gyro scale. The observer makes sufficient observations, about 1-2 hours, to determine the mid-point of swing for each spin and non-spin mode of the spinner. The method makes use of the maximum data available from the Gyrotheodolite.

The method is very simple and safe to be used for practical application in surface and subsurface baseline orientation, for example, for azimuth determination in underground tunnels. Precautions need to be considered for the stability of the equipment and protection from sunshine and wind when used on the surface. However, additional precautions need to be considered if this method used in a mine, the system must be acceptable for mine safety for electrical equipment.

4. Gyrotheodolite adjustment computations:

4.1 Mathematical Model:

The mathematical model, eq. (1) represents the mathematical relationship between *X* the unknown parameters and *L* the observations. It may be written as in Vanicek (1982) as:

$F(X, L) = 0$

$$F(X, L) = \sum[A_i e^{-\lambda i t} \cos((\theta_i + \theta_i' t)(t - B_i))] + Q - \Delta = 0 \qquad i = 1, \ldots 10$$

$$X = (A, \theta, \theta', B, \lambda, Q)$$

Suppose the scale divisions are not equally spaced. This will depend on the manufacturer's precision. Therefore, they are considered as observable quantities:

$$L = (t_1, \Delta_1, t_2, \Delta_2 \ldots\ldots\ldots t_n, \Delta_n)$$

Where n is the number of observations.

4.2 Weight of Observations:

The observations are time and scale divisions at each moment the moving mark crosses a scale division. The correct weighting of each observed time and scale division may be given as a function of either the distance from the midpoint of swing or the velocity of the moving mark. However, Breach (1983) found that taking different weights had no effect on the determination of the centre of oscillation Q or on its standard deviation σ_Q. Therefore, all observations should have equal weight.

4.3 Some Computing Considerations:

Least squares adjustment techniques are used to compute the different parameters for eq. (1). Since the initial approximate values of the parameters are not known, good provisional values of the unknown parameters are used to get a convergent solution by an iterative least squares process. Since some parameters are numerically very much larger than others are, setting all parameters to zero, as initial values did not work. The problem of finding the provisional values of the parameters consists of the following steps:

- Put $\theta' = 0$ in eq. (1), compute the provisional values of the parameters by non-rigorous means.
- Put $\theta' = 0$ and $\lambda = 0$ in eq. (1), compute the values of the parameters by iterative least squares using the provisional values derived from the previous step.
- Put $\theta' = 0$, use the values obtained in the previous step to compute all the values of the parameters in eq. (1) up to the next value of i by iterative least squares.
- Repeat these three steps to compute the values of the parameters in eq. (1) up to the next value of i by iterative least squares.

5. Results and conclusions:

A new mathematical model for the motion of the moving mark of the Gyrotheodolite is found and can be used in many practical applications. The maximum available data were used with the aid of video camera. The accuracy of timing method increased considerably. The midpoint of swing was determined at a high precision. As a result, a high precision of azimuth determination was obtained. Values of eq. (1) parameters are summarised in table 2:

Table 2 Values of parameters A_i, θ_i, θ'_1, λ_1 and the periods T_i

i	A_i arc seconds	θ_i rad./sec.	θ'_i rad./(sec.)^2	λ_i unitless	period in seconds $T_i = 2\pi/\theta_i$ when $t = 0$
1	6596.68	0.0148	5.05E-11	1.01E-07	424.54
2	14.82	0.016525			380.22
3	13.58	0.013225			475.10
4	0.46	0.137325			45.75
5	0.75	0.1247			50.39
6	3.13	0.01945			323.04
7	1.25	0.022025			285.27
8	1.17	0.018475			340.09
9	0.88	0.027275			230.36
10	1.49	0.02445			256.98

Table 3 Values of ***Q*** and their standard deviations

First set of observations			Second set of observations		
i	Q	σ_Q	i	Q	σ_Q
1	-0.1028	0.00113	1	-0.9358	0.00243
2	-0.1049	0.00062	2	-0.9357	0.00244
3	-0.1044	0.00038	3	-0.9357	0.00244
4	-0.1045	0.00038	4	-0.9359	0.00243
5	-0.1043	0.00037	5	-0.9357	0.00218
6	-0.1043	0.00034	6	-0.9365	0.00166
7	-0.1043	0.00034	7	-0.9365	0.00166
8	-0.1044	0.00034	8	-0.9366	0.00166
9	-0.1044	0.00034	9	-0.9366	0.00167
10	-0.1044	0.00033	10	-0.9366	0.00167

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