1D and 3D Analyses of Deformations in Engineering Structures Using GPS and Terrestrial Data

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SUMMARY

This paper discusses the augmentation of carrier-phase differential GPS (DGPS) with terrestrial levelling to assess the accuracy improvements possible in both vertical and 3D components when applied to deformation monitoring of a viaduct. The observations are combined using Minimum Norm Quadratic Unbiased Estimate (MINQUE) methods. This paper reports on initial findings in a study on the combination of varying densities of levelling data with DGPS results, and the effects that the constraint of this higher accuracy method have on the whole network.

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1. INTRODUCTION

In the past, DGPS has been used in deformation analysis to determine parameters at the centimetre level, or even at the millimetre level in some cases (Beutler et al., 2001; Hartinger and Brunner, 1998). Once differential and site-specific errors have been eliminated or reduced, the accuracy of the method for differential heighting may still be inappropriate for a specific measurement task, however. In the past, combining different observation types (geodetic or otherwise) has been used to overcome the disadvantages of one system in a particular situation, for instance Chrzanowski and Chrzanowski (1995). In DGPS levelling, the variance of height is normally the highest component of the 3D solution due to receiver-satellite geometry and residual path-delay problems. This is important due to the reliance on estimated variance in the analysis of global and local deformation using Fisher difference testing. This study reports on the benefit of combining different levelling strategies with DGPS levelling to improve the estimated variance of the differential heights in a deformation network. Precise levelling is also used as the basis of an external accuracy check for these combined networks. A discussion is included on further work that will use such augmentation of DGPS networks to potentially improve planimetric accuracy.

2. FIELD WORK

In this study, the deformations of the Karasu viaduct (Figure 1) were investigated using GPS and precise levelling data. Karasu viaduct is, at 2160 m, the longest viaduct in Turkey. It is located to the west of Istanbul and forms a part of the European Transit Motorway. The first 1000 meter of this viaduct crosses over the Büyükçekmece Lake and some of the piers of the structure were constructed in this lake (see Figure 2). The viaduct carries a dual two-lane road and was constructed on 110 piers (55 piers north and south). There is 40-meter width between two piers, and one deformation-monitoring point is constructed within every fifth pier.

Figure 1: The Karasu viaduct

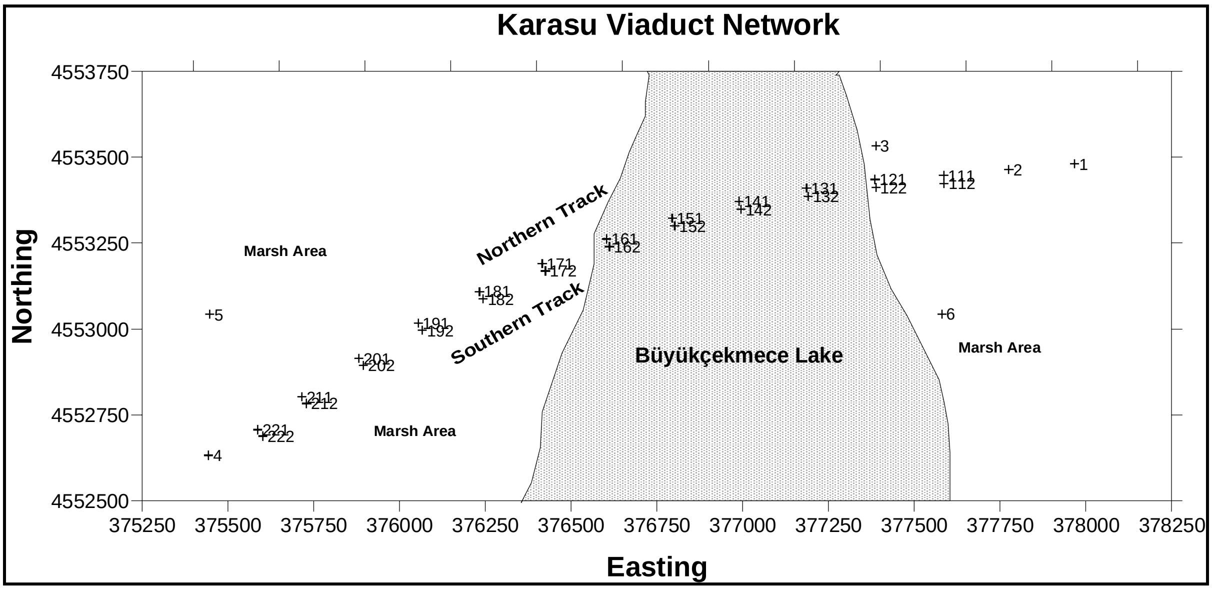


Figure 2: Schematic of the viaduct showing the location of the piers

The deformation measurements of Karasu involved four measurement campaigns carried out in June 1996, March 1997, October 1997 and April 1998. These four campaigns include GPS measurements and precise levelling measurements. A local geodetic network had been established using GPS techniques (control stations 1-6 in Figure 2).

In the four GPS campaigns, up to eight receivers were deployed at once (see Table 1 for a list of receiver / antenna combinations used) each for an occupation period of 4 hours, with a second occupation made for each survey monument. The baselines were approximately 2km in length. In addition to the six control stations, 24 deformation points on the viaduct were measured in each campaign as shown in Figure 2. At the same time a Koni 007 level was used to observe differential heights between the stations.

Table 1: GPS receiver/ antenna combinations used in the project

Type	Receivers	Antennas
1	LEICA SR399	LEICA AT202/302 (LEIAT302-GP = SR299/SR399 Ext. w/o g.p.)
2	LEICA SR399	LEICA INTERNAL (LEISR399_INT = SR299/SR399 Int. Antenna)
3	LEICA SR9500	LEICA AT202/302 (LEIAT302-GP = SR299/SR399 ext. w/o g.p.)
4	TRIMBLE 4000SSI	COMPACT L1/L2 GND (TR GEOD L1/L2 GP = TRM22020.00+GP)

3. DATA PROCESSING

GPS data processing for each campaign used the Leica SkiPro software, with parameters specified as in Table 2.

Table 2: Processing strategy for the GPS campaigns using SkiPro

Parameter	Setting	Description
Cut-off angle	15 degree	
Sampling	15 second	
Ephemeris	Precise (IGS)	
Solution type	Automatic	In Automatic processing SKI-Pro automatically selects the best frequency or combination of frequencies for the final solution.
Tropospheric model	Hopfield	
Ionospheric model	Automatic	SKI-Pro selects a model to be used according to the duration of the sessions user involvement. For observation times on the reference longer than 45 min. your own ionospheric model may be computed, so that automatically the option Computed model will be taken, whereas with shorter observation periods the Klobuchar model will be preferred. If no almanac is available, though, No model will be used with observation times below 45 min.
Phase Centre Correction	IGS file	

4. COMBINING GPS AND LEVELLING RESULTS

4.1 Variance Component Estimation

The relative weights applied to the observations in a network solution affect the parameter estimate outcomes. In addition, the magnitude of the weights, once corrected according to the *a posteriori* reference variance (if deemed appropriate after hypothesis testing) affects the estimated variances of the parameters. It has been recognised that the most appropriate weighting scheme for GPS double-differencing is not a diagonal matrix of the same weights, due to the statistical correlation between the double-difference observations (Wang *et al*, 1998). In that case, Minimum Norm Quadratic Unbiased Estimation (MINQUE) was used to find the most appropriate individual weights for each observation iteratively, based on the full variance-covariance (V-C) matrix of the double-differenced observations. In this study MINQUE has been used to provide an *a priori* weighting scheme for GPS-baseline network solutions by combining the V-C matrix of levelling observations with the V-C matrix of the GPS observations, to examine how this will improve the network results.

The general theory and algorithms of the minimum norm quadratic unbiased estimation procedure are described (Rao and Mitra, 1971; Rao, 1971; Rao and Kleffe, 1988). MINQUE is classified as a quadratic-based approach where a quadratic estimator is sought that satisfies the minimum norm optimality criterion. Given the Gauss-Markov functional model $v = Ax-b$, where v and b are vectors of the observations and residuals, the selected stochastic model for the data is derived as follows:

$$C_b = \sum_{i=1}^{k} \sigma_i^2 Q_i = \sigma_1^2 Q_1 + \sigma_2^2 Q_2 + ...+ \sigma_k^2 Q_k \qquad (1)$$

$$C_b = \text{diag}[\sigma_i^2 Q_i] = \begin{bmatrix} \sigma_1^2 Q_1 & 0 & 0 & 0 \\ 0 & \sigma_2^2 Q_2 & 0 & 0 \\ 0 & 0 & .... & 0 \\ 0 & 0 & 0 & \sigma_k^2 Q_k \end{bmatrix} \qquad (2)$$

Where only the variance components σ_i^2 for k observations are to be estimated. The MINQUE problem is reduced to the solution of the following system

$$S\sigma_i^2 = q \ (3)$$

S is a *k*k* symmetric matrix that may not be of full rank and therefore its pseudo-inverse can be used for solving Eq. (16). Each element *Sij* in the matrix S is computed from the expression

$$S_{ij} = tr(RQ_iRQ_j) \ ,i,j = 1,2,\ldots.k \ (4)$$

where *tr( )* is the trace operator, *Q( )* is a positive definite cofactor matrix for each group of observations. *R* is a symmetric matrix defined by

$$R = C_b^{-1}(I-A(A^TC_b^{-1}A)^{-1}A^TC_b^{-1}) \ (5)$$

where *I* is an identity matrix , **A** is an appropriate design matrix of full column-rank and *Cb* is the covariance matrix of the observations. The vector *q* contains the quadratic forms

$$q = \{q_i\}, \qquad q_i = v_i^T Q_i^{-1} v_i = b^T RQ_iRb \qquad (6)$$

where *vi* are the estimated observational residuals for each group of observations *bi***.** As a result we can generate the Eq. (7) as below.

$$\begin{bmatrix} tr(RQ_1RQ_1) & tr(RQ_1RQ_2) & ...... & tr(RQ_1RQ_k) \\ tr(RQ_2RQ_1) & tr(RQ_2RQ_2) & ...... & tr(RQ_2RQ_k) \\ ...... & ...... & ...... & ...... \\ tr(RQ_kRQ_1) & tr(RQ_kRQ_2) & & tr(RQ_kRQ_k) \end{bmatrix} \begin{bmatrix} \sigma_1^2 \\ \sigma_2^2 \\ .... \\ \sigma_k^2 \end{bmatrix} = \begin{bmatrix} b^TRQ_1Rb \\ b^TRQ_2Rb \\ ...... \\ b^TRQ_kRb \end{bmatrix} \qquad (7)$$

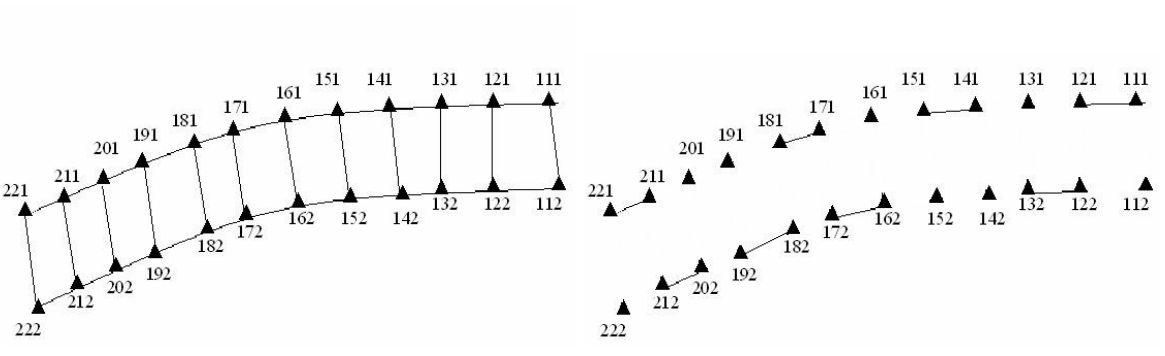
The computed values from a first run through the MINQUE algorithm is $(\sigma_i^2)_1$. $(\sigma_i^2)_0$ can be specified *a* priori as unity, for each variance factor. The resulting estimates $(\sigma_i^2)_1$ can be used as 'new' *a priori* values and the MINQUE procedure repeated. Performing this process several times is referred to as iterative MINQUE (IMINQUE). The iteration is repeated until all variance factor estimates approach unity. The final estimated variance component values can be calculated by

$$\sigma_i^2 = \prod_{\alpha=0}^{n} (\sigma_i^2)_\alpha$$

Where n is the number of iterations in the MINQUE evaluation.

4.2 Combining the Levelling Observations with the GPS Observations using MINQUE

Figure 3 shows the level network between the monitoring points on the viaduct. The links to control points 1-6 are not shown.



(a) (b)

Figure 3: Schematic view of the monitoring points and level lines included in the combined network solution for the full set of levels (a) and the 8 level observation set (b)

Seven networks were adjusted using different numbers of differential heights from optical levelling in addition to the full DGPS data set. The number of differential heights included varied from zero (therefore using just the 39 GPS baseline solutions) to 40. To generate the different levelling sets, the original 40 levels were decimated so that each set was distributed across the network, rather than having clusters of level observations in any particular area (Figure 3). The MINQUE method was used to design *a priori* weights for the combined observations using the variances estimated for the observations during their individual adjustments.

4.3 Results of the Experiment

Figure 4 shows the results of the inclusion, using MINQUE to derive *a priori* weights, of varying numbers of differential levels into the network solution. The polynomial curve describes the relationship between the number of differential levels and the mean precision of differential height estimation, judged by taking the full level network solution as ground truth. Figure 5 displays the estimated standard deviation for the network solutions. It can be seen that using zero differential levels (GPS-only solution), the estimated standard deviation is optimistic. At the other end of the chart, with the full network of level observations included, the estimated standard deviation is slightly pessimistic. Whilst a clear benefit is derived from including even a relatively small set of level observations (improvement from 6.5mm to 3mm using 8 differential levels in Figure 4), there is no corresponding change in estimated variance using the MINQUE method. This is, of course, a problem for any deformation analysis based on estimated variances.

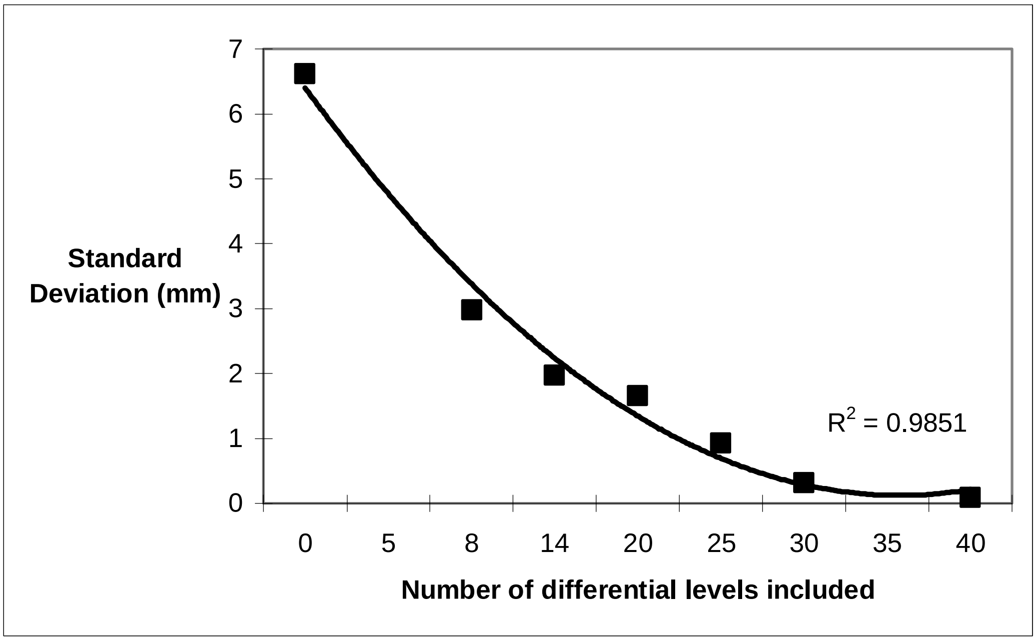


Figure 4: Pooled standard deviation (compared to the full level network) versus number of differential level observations included in the network solution

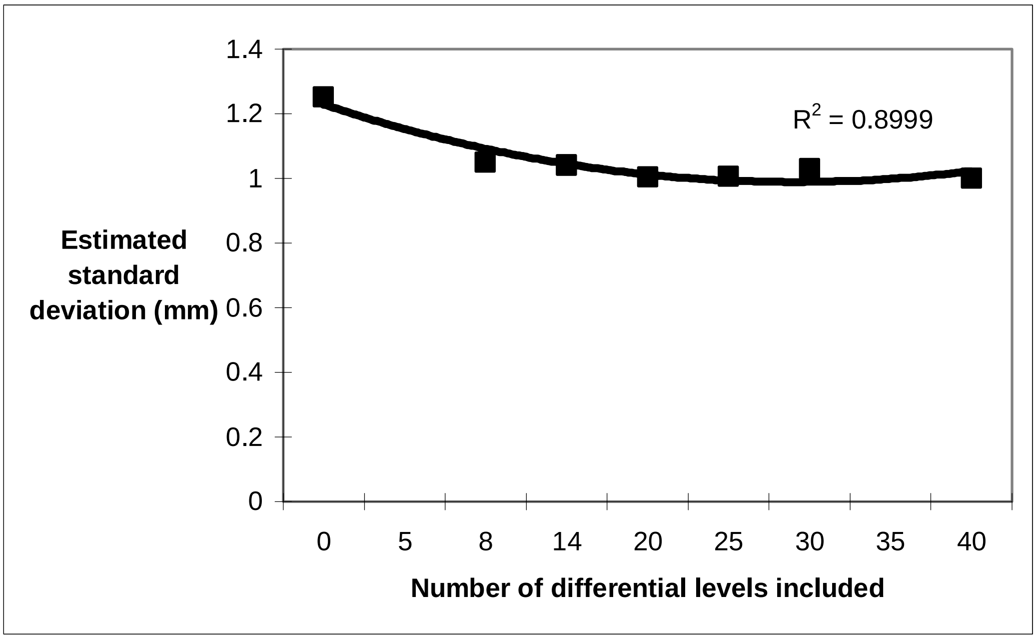


Figure 5: Pooled estimated standard deviation versus number of differential level observations included in the network solution

5. CONCLUSIONS AND FURTHER WORK

This paper has reported initial findings from an investigation into the effects of augmenting DGPS levelling with precise optical levelling observations. The MINQUE method of variance component estimation has been used to derive *a priori* values for the weights of combined observation types. Benefits in the height accuracy of the resulting network have been seen with even a small set of levels. However, the estimated variances derived from the network solution are not reflective of the range of improvements achieved using sequentially greater numbers of additional observations.

Further work on this subject will focus more closely on the statistical analysis of the network when using such combined observations and a MINQUE weighting scheme. Specifically, the question will be examined of why the variances for the various network solutions do not differ significantly. Additionally, it will be investigated whether or not networks from different epochs with different numbers of additional observations (say, 10 levelling observations and 20 levelling observations, respectively) are drawn from the same distribution. The latter is a fundamental pre-requisite to the methods of congruency testing, and is usually checked using Bartlett's test, or alternatively the pooled t-test. The potential for the levelling observations to improve planimetric accuracy in GPS network solutions using *a priori* weights derived from MINQUE will also be investigated.

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