OPTIMAL DESIGN OF DEFORMATION MONITORING NETWORKS USING PSO ALGORITHM

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Abstract: Geodetic deformation monitoring networks has to be sufficient in terms of precision, reliability and strength, sensitivity and cost. Hence, before monumenting and gathering survey data, a geodetic network must be designated to meet some quality criteria. Mathematically, optimal design of a geodetic network implies minimizing or maximizing an cost function that denotes the quality of the network. Classically, a network can be optimized using the trial and error method or analytical methods such as linear programming, quadratic programming, or some optimization problems can be solved by generalized or iterative generalized inverses. Optimization problems may also be solved by intelligent optimization techniques such as genetic algorithms, simulated annealing and particle swarm optimization algorithm (PSO). In this paper, we dealt with optimization problem of geodetic networks. A GPS network was optimized by using PSO algorithm in the sense that it will satisfy good precision and low cost requirements. According to our results, PSO algorithm can be used as a tool for optimizing a geodetic network.

1. INTRODUCTION

Optimization implies minimizing or maximizing an objective function which expresses the criteria adopted to define the quality of the network. Generally, the quality of a geodetic network have been characterized by its precision, reliability and strength, and economy. But, one more criterion are added to these criteria for deformation monitoring networks, that is sensitivity criterion. Precision is a measure of the network's characteristic in propagating random errors. The main purposes of the present contribution is to design and optimize of a GPS network in the sense of high precision and low possible cost. Grafarend (1974) classifies different optimization problems into different orders, that is:

a) zero-order design (ZOD): optimum datum definition
b) first-order design (FOD): design of the optimum network configuration
c) second-order design (SOD): selection of the optimum observational weights
d) third-order design (THOD): improving an existing network by adding extra points and/or observations

Sometimes the FOD and the SOD design problems can be solved simultaneously. In this case, the design problem is called a combined (COMD) problem (Kuang, 1996).

Traditionally, geodetic network optimization problems can be solved using either trial and error method or analytical methods. Unfortunately, these classic approaches can give rise to some problematic cases. For example, when the trial and error method are used, optimal network may never be found and a great quantity of computation may be required. Similarly, analytical methods may produce absurd solutions such as negative weights or disconnected networks. Furthermore, the planned network may never be achieved (Kuang, 1996). On the other hand, local optimization techniques such as Kuang's methodology in the geodetic literature can be converged to any local optimum instead of global optimum. Can the optimal design problems of geodetic networks be solved using a more simple and efficient method?

The main purposes of the present paper is to realize of second-order design of a GPS network that can be used for deformation monitoring in the sense of desired precision and low possible cost using PSO method. That will provide an optimum survey planning and prevent unnecessary observations.

2. OPTIMAL DESIGN OF GEODETIC NETWORKS

In the literature of geodetic network optimal design, optimization means minimizing or maximizing of an objective function that represent the goodness of the network. The goodness of a geodetic network can be measured by precision, reliability and strength, and cost. Different objective functions reflecting these criteria can be used in the optimization procedure. Only precision and cost criteria are considered in this paper. Criterion matrices are very adequate tools to set up objective function. They represent a desired precision for the network results. If a criterion matrix is used in the optimization procedure the following objective function can be employed:

$$\left\| C_x - C_s \right\| \rightarrow \min. \tag{1}$$

In all of the optimization problems, the main task is to find optimization variables by minimizing or maximizing the chosen objective function. Furthermore, the values of optimization variables can be restricted according to some specific constraints. Now, we will discuss these two important concepts of optimization procedure the second-order design of geodetic networks point of view, namely, optimization variables and constraints.

In the SOD, the P matrix of observation weights is the optimization variables. On the other hand, the A matrix represents the geometry of the network. If both matrices are known, the covariance matrix for the unknowns in the adjustment problem is given by

$$C_x = \sigma_0^2 \left(A^T P A \right)^{-1} \tag{2}$$

As is well known, covariance matrix of the unknown parameters contains complete information about the precision of a geodetic network.

The weights of observations should be non-negative and be bounded by the maximum achievable accuracy of the available instrument(s), i.e.,

$$0 \le \mathrm{P_i} \le \frac{\sigma_0^2}{\left(\sigma_\mathrm{i}^2\right)_{\min}} \tag{3}$$

where σ_0^2 is the a priori variance factor and $\left(\sigma_i^2\right)_{\min}$ are the minimum variances that can be achieved for each observation (Kuang, 1996).

As mentioned above, classic methods that appeared in the literature may cause some problematic cases. Recently, many optimization problems have been solved by using techniques of artificial intelligence. These techniques are also named natural optimization methods. Examples of natural optimization techniques are simulated annealing (Kirkpatrick et al., 1983), genetic algorithms (Haupt and Haupt, 2004) and PSO (Parsopoulos and Vrahatis, 2002). These techniques emulate optimization processes encountered in the nature. For example, PSO mimics collective behavior of some creatures such as birds and bees. In the next section we will discuss the PSO method.

3. PARTICLE SWARM OPTIMIZATION

PSO, which is an iterative-heuristic, population-based search algorithm, is proposed by R.C. Eberhart and J. Kennedy in 1995. It emulates collective intelligence of bird flocking, fish schooling and bee swarming to converge to the global optimum. In the frame of PSO, a swarm consists of interacting agents that is particles. The characteristics of the particles depend on the problem of interest. Collaboration among the particles provides global optimum to the problem. These particles move in an *D*-dimensional search space, in an attempt to discover ever-better solutions.

Each particle of the swarm has a current position vector and an adaptable velocity vector. Position vector contains optimization variables. For example, in this study optimization variables are observation weights. Furthermore, each of the particles has a memory. During the iterative procedure, they remember both the best position found so far by each particle of the swarm and the best position found so far by all the particles. At each iteration step, particles are shifted from their current position by applying a velocity vector to them. As it is clear, the velocity of each particle have been updated at each iteration of the algorithm.

The manipulation of the swarm have been implemented according to following two equations:

$$v_{i+1} = C\left(wv_i + c_1 r_1\left(p_i - x_i\right) + c_2 r_2\left(p_g - x_i\right)\right) \tag{4}$$

$$x_{i+1} = x_i + v_{i+1} \tag{5}$$

Here, the position of the *i*th particle is represented as $x_i = (x_{i1}, x_{i2}, ..., x_{iD})$, and the velocity for *i*th particle is represented as $v_i = (v_{i1}, v_{i2}, ..., v_{iD})$, the best previous position of the *i*th particle is represented as $p_i = (p_{i1}, p_{i2}, ... p_{iD})$, p_g is the best solution achieved so far by the whole swarm. In Equation 4, there are some parameters that must be explained. r_1 and r_2 are random numbers, uniformly distributed in [0,1]. They are used to add a stochastic element to the movement of the particles. Constants c_1 and c_2 determine the balance between the influence of each particle's knowledge (c_1) and that of the whole swarm (c_2). These constants are called cognitive parameter and social parameter, respectively. C is constriction factor, which is used to limit velocities. The velocity of the previous iteration is kept weighted with w, i.e., the inertia weight. The inertia weight and constriction factor prevent the algorithm to converge on premature solutions.

Finally, the basic strategy for the implementation of the PSO algorithm is given as follows:

1. Initialization

   j=0
   (a) Determine the objective function, optimization variables and constraints
   (b) Select PSO parameters such as inertia weight, constriction factor and social and cognitive parameters
   (c) Select neighborhood topology
   (d) Randomly generate initial particle positions x_i^0 in D-dimensional search space
   (e) Set initial particle velocities to zero; $v_i^0 = 0$
   (f) Set j=1

2. Optimize

   (a) Evaluate objective function value f_i^j using particle positions x_i^j
   (b) If $f_i^j \le f_i^{best}$ then $f_i^{best} = f_i^j$ and $p_i = x_i^j$
   f_i^{best} is the particle's personal best cost
   (c) If $f_i^j \le f_{global}^{best}$ then $f_{global}^{best} = f_i^j$ and $p_g = x_i^j$
   f_{global}^{best} is the best cost of whole swarm
   (d) If stopping criterion is satisfied then go to 3
   (e) Update all particle velocities v_i^j by Eq. (1)
   (f) Update all particle positions x_i^j by Eq. (2)
   (g) Increase j
   (h) Go to 2(a)

Stop

For more detailed information on PSO, interested readers refer to Kennedy and Eberhart (2001), Clerc and Kennedy (2002), Eberhart and Shi (2000) and Parsopoulos and Vrahatis (2002)

4. NUMERICAL EXAMPLE

The second-order design of GPS networks using classic operation research methods was investigated in Kuang (1996). To demonstrate the applicability of PSO to the SOD of a GPS network an example is provided below. If a set of points whose relative coordinates to be estimated by GPS relative positioning technique, a list of possible baselines that can be measured in the field and the precision criteria for the estimated coordinates are given, PSO searches for the optimal set of observational weights and their corresponding observational precisions.

Figure 1 depicts a GPS network consisting of 4 points and 6 baselines. This network can be used for deformation monitoring. The desired precision can be described by a criterion matrix. In our example, we used the following criterion matrices:

$$C_s = diag\{1^2 \quad \dots \quad 1^2\}(mm^2) \tag{6}$$

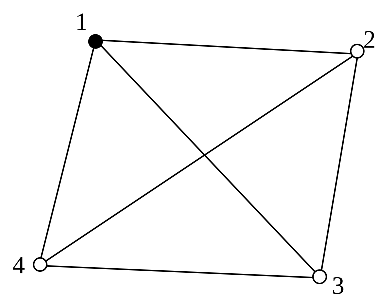

Figure 1- GPS Network

Let us perform a SOD following the objective function expressed in Equation 1. PSO was used as a solution strategy. Chosen parameters for PSO are listed in Table 1.

The maximum and minimum weights are define the search space, i.e., particle position are restricted with minimum and maximum weights. Maximum weights are calculated using the precision of available instruments. The minimum and maximum weights are shown in the columns 3 and 4 of Table 2, respectively.

Parameter	**Value**
Particles	30
Iteration	100
c_1 and c_2	2.05
w	decreasing from 1 to 0 during the iterative process
C	0.729

Table 1- PSO parameters

The obtained optimization results using PSO method are summarized in Table 2, Table 3 and Table 4.

From	To	P(min)	P(max)	P(opt.)
1	2	0	1	0.7423
1	3	0	0.5	0.4549
1	4	0	1	0.7126
2	**3**	**0**	**1**	**0.3948**
2	**4**	**0**	**0.5**	**0.0000**
3	4	0	1	0.4816

Table 2- Optimization Results

As can be seen from Table 2, the optimal weight for the baseline of 2-4 baseline is zero. Accordingly, this baseline are eliminated from final observing plan. If the baseline of 2-3 is eliminated due to its optimal weight is insignificant compared to other baselines, final results for the network are obtained as given in Table 3. The optimal weights of the rest baselines are replaced by maximum weights given in the column 4 in Table 2, because maximum weights are calculated with respect to the precision of available instruments. In Table 3, the variances of the coordinates have been shown. Since the correlations among all GPS baseline components are neglected, the standard deviations of the coordinates are the same for any point. According to these results, our criteria given in Equation 6 are satisfied for all points.

Point	σ_x^2	σ_y^2	σ_z^2
2	1	1	1
3	1	1	1
4	0.75	0.75	0.75

Table 3- The variances of the coordinates after optimization

5. CONCLUSION

An optimal network in the sense of desired precision and low possible cost can be achieved with the PSO algorithm. The main goal of the present contribution was to solve SOD problem in a GPS network in order to find optimum observations accuracy. Having applied the PSO algorithm to the problem, the observations that is obtained with zero weight are removed from the observing plan.

It should be noted that the application of the PSO algorithm to the geodetic optimization problems are very preliminary. For example, different optimization problems such as the FOD of a geodetic network can be solved using this technique, or sensitivity criterion can be dealt with for deformation monitoring networks. On the other hand, some studies about the characteristics of the PSO algorithm may be investigated, to illustrate the proper selection of PSO parameters can be searched. Furthermore, since the PSO is a stochastic method, initializing the algorithm and producing random elements of the algorithm can be examined in detail.

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