Accuracy Aspects of Processing and Filtering of Multibeam Data: Grid Modeling versus TIN Based Modeling

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Key words: Multibeam data, digital elevation model, quality assessment, filtering

SUMMARY

Multibeam echosounder measurements serve to make a digital terrain model of the seafloor. The Delaunay triangulation is a widely appreciated and investigated mathematical model to represent the sea bottom topography and is highly efficient for building TINs (Triangular Irregular Networks) out of non-homogeneous data such as raw multibeam data.

Obtaining an accurate model of the sea floor is a major concern in dredging works. Contemporary hydrographical surveying tools, especially the multibeam echosounder, yield a very dense point sampling of the seafloor. Subsequently, this immense amount of data needs to be processed in order to generate an accurate terrain model, according to time- and accuracy constraints imposed by the client. Modeling can be carried out in post-processing or in real-time. Performing a real-time accountability keeps track of the cur- or fill volume changes realized at that moment. Most multibeam systems deliver equidistant interpolated data, allowing faster processing to be achieved using equidistant grid-based modeling.

Both modeling techniques (TIN and grid) yield their own advantages and drawbacks. More specifically, the filtering options of TIN and grid models are quite different. In this paper, the accuracy of volume computations in a TIN is mathematically derived on a statistical basis.

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1. INTRODUCTION

Obtaining an accurate model of the sea floor is a major concern in dredging works. Nowadays' hydrographic surveying tools, especially the multibeam echosounder, yield a very dense point sampling of the sea floor. The immense amount of data needs to be processed (e.g. filtering or data reduction), to form an accurate terrain model according to the constraints imposed by the client. Modeling can be performed in post-processing or in real-time. Performing a real-time accountability will keep track of the haul, realized at a specific moment.

DTM (Digital Terrain Model) software for hydrographic purposes must meet the following four requirements:

1. Fast model creation: the purpose is to create the model as fast as the data is gathered, so that real-time control and verification are possible;

2. Manual editing of the model: adding data points as well as deleting data points (vertices) in the model are both required. When examining the theoretical model of a site, intervening directly in the model as it is displayed on the computer monitor by relocating, deleting or adding vertices is a prerequisite. It should also be possible to replace data from resurveyed areas with more recent data and to update the existing model with this new information;
3. Data reduction: reducing the large amounts of multibeam data to acceptable levels, keeping the sea floor model as accurate as possible, but the data set manageable for the used computers;
4. Data quality: the final result in the form of volume calculations should be as close to the truth as possible, and certainly not further away than acceptable, assuming that the acceptable quality level is realizable. In order to qualify the data, different statistical tools can be applied (Höhle & Höhle, 2009). The final data model and the derived volume computation should give a correct approximization of the real situation. The acceptability of the model is related to the measurement device, as well as to the requirements of the client.

Grid models and triangulation models (TINs) are the most frequently used models in hydrography (Brouns *et al.*, 2001), offering different kinds of advantages and drawbacks. Both terrain representations are discussed with their advantages and drawbacks, with attention put to the filter capabilities of each approach. Grid based filtering approaches (Wack & Wimmer, 2002) and TIN based filtering approaches are intensively discussed in the field of airborne laser scanning, using similar approaches. The distinction between TIN based and grid based filtering techniques can be made by respectively an original point set filtering and a filtering based on interpolated equidistant cells.

In general, filtering techniques can be divided in four categories (Krzystek, 2003):

- Least squares interpolation (Briese & Kraus, 2003);
- Local slope based filtering (Vosselman, 2000; Sithole, 2001);
- Morphological filters (Zhang *et al.*, 2003) ;
- Convex hull filtering Krzystek, 2003).

2. GRID MODELING

2.1 Principle

Nowadays, most multibeam systems offer equidistant grid data as default output of the on-line and on-board processing chain. The plane coordinate system used is generally a square grid with the axis parallel to the Easting and Northing axis of the grid coordinate system used. Since the use of GNSS equipment, the universal transverse Mercator system (UTM) in relation to the ETRS89 datum (referencing the global ellipsoid GRS80) has established itself as the standard grid system in Europe. Height/depth values can be related to a universal reference (e.g. the WGS84 ellipsoid, LAT, geoid…) or, in most cases, to a conventional reference plane. In the latter case, if the GNSS receiver on board the vessel gives the height above the WGS84 ellipsoid, a conversion matrix between ellipsoid and the reference plane used shoud be given. Alternatively, the older technique of tide gauges can be used, where the measured depths are related to the water surface, and the water surface related to the reference plane by means of tide gauges. This leaves the grid interval distance as the unique and most important user-defined parameter.

The use of equidistant points allows storing only the depth values in computer memory and not the Easting and Northing values, since these values can be computed out of the row and column number of each point, assuming (for instance) that the point storage is performed in a row-wise manner in the computer memory. Memory use can be optimized by using arrays of integer values that, for dredging based surveys, can be limited to only 2 byte for each depth or point, giving a range of 2^{16} or 65536 height/depth values, or a range of approximately 65 m with the mm as unit. An example of this file structure is implemented by ESRI (ESRI, N.N.).

2.2 Filtering

As the amount of data generated by a multibeam echosounder depends on the ping rate, which goes up to 30Hz, and the number of beams in the swath, typically between 100 and 500, incoming data flows can reach up to more than 50 million points per hour. It will be clear that reducing the data acquired by multibeam echosounding is indispensable; because of the huge amount of data and since most of the measured points do not contribute to a more detailed sea floor approximation. An ongoing concern is therefore dataset reduction.

Data set reduction can be performed by the multibeam software itself by creating equidistant cells out of the mass of measured points. Moreover, extra data reduction by the user is possible by increasing the grid interval distance, either in the software of the multibeam manufacturer, or in the software used for the post-processing. While the first one often works as a black box without much control on the reduction parameters used, the use of post-processing software offers a number of user-defined filter-processing options.

One frequently used type of filtering is to increase the grid interval distance, e.g. an initial 0.2 m by 0.2 m grid is reduced to a 1 by 1 m grid, yielding a reduction factor that is the square of the linear proportions, which is a factor of 25 in this example. For the filtering algorithm, different approaches are possible to compute the elevation value in the resulting grid cell with increased resolution:

- The depth is the average of all depths of the initial grid points lying inside the resulting bigger grid cell;
- The depth is a weighted average of all depths of the initial grid points lying inside the resulting bigger grid cell. A weighting factor is used, which is frequently the inverse of the distance between each initial grid point and the average, raised to the power n. Often, n = 2 is chosen, yielding an inverse quadratic distance as weighting factor;
- The depth can be taken to be the "shallowest" value of all depths of the initial grid points lying inside the resulting bigger grid cell. This can be motivated if the purpose is to determine the minimal sea bottom depth, rather than computing an accurate volume, as can occur in dredging projects. Analogously, in reclamation projects, the "lowest height" can be the most important characteristic of each resulting grid cell.

2.3 Advantages and drawbacks of grid models

The principal advantages of grid models are the simplicity of these basic models and the low memory requirements for the processing of the depth data, since planimetric coordinates are computed afterwards, and not stored in the memory. Hence, computations are fast and quite straightforward. In the case of homogeneous sea bottom coverage by a multibeam sensor, grid models are often the preferred data model for the bathymetric modeling of the sea bottom. Due to the less complex algorithms, involved in the computational geometry modeling operations, real-time modeling is easier to implement using grid modeling than using TIN modeling.

An obvious drawback of grid systems is the loss of the initial measured bathymetric survey points. This information is used to interpolate the depths of the grid points, yielding a planimetric shift of the data with depth information and resulting in a global smoothing of the digital sea bottom model. Moreover, the used interpolation technique will result in an additional error on the data, as a function of the point density, local curvature and the used interpolation technique (Chaplot *et al.*, 2006). This can be particularly frustrating when a relatively small object, with important depth variation is measured, for example a sleeve for pipe-laying projects. Typically, the sleeve width has to be realized within decimeter range accuracy. When a high density grid model with an interval distance of 1 meter is used, the sleeve design will be highly distorted. This can be counteracted by the use of a heterogeneous model with variable grid intervals depending on the area. Quadtree structures can be used in these cases of modeling. However, heterogeneous models are complex, involving data manipulation routines that are difficult to implement and require significant higher amounts of computer memory and processing time, in comparison with regular spaced grid models.

3. TIN BASED MODELLING

3.1 Principle

It is a common practice to use the Delaunay triangulation (Brouns *et al.*, 2003) to construct a TIN rather than other, less restrictive triangulations. In a Delaunay triangulation, the circumscribing circle of any triangle contains no other vertices (Shewchuck, 1996).

Triangles whose circumcircle does contain another vertex are invalid and need to be replaced by another triangle by a process called *edge flipping*; this principle is shown in Figure 1 (left, middle). The triangles *ABC* and *ACD* are not Delaunay triangles, since they contain *d* and *b* respectively in their circumscribing circles. After flipping the edge *ac* to *bd*, the triangles *ABD* and *BCD* are created, which do not contain other vertices in their circumscribing circle. They therefore meet the Delaunay requirement.

Figure 1 (right) represents what is called *edge completion*: when four points are co-circular, the resulting quadrilateral is (arbitrarily) split in two separate triangles. This constitutes a degenerate case as either of the two diagonals can be constructed.

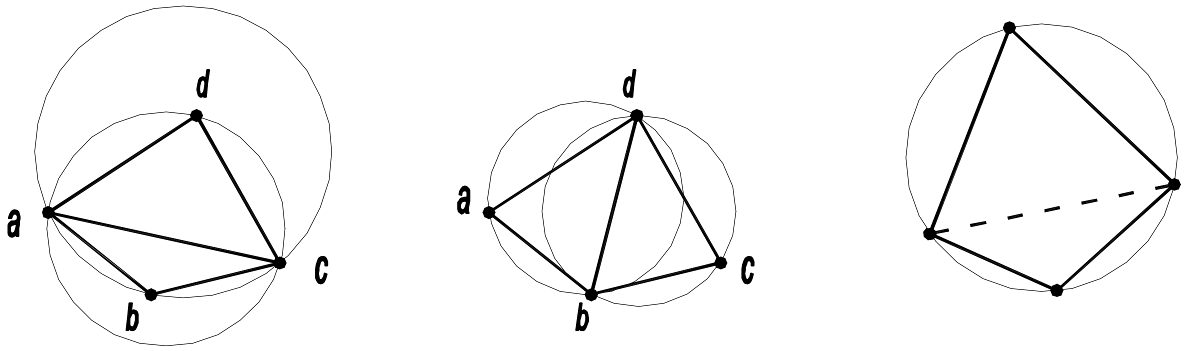


Figure 1: Delaunay triangle principle

It can be proved (Shewchuck, 1996) that the Delaunay triangulation of a set of vertices is unique; this is an important quality asset towards the client, as it allows him to repeat the calculations to verify the results independently.

3.2 Filtering

A necessary feature for a bathymetric survey program is editing, in order to optimize the digital terrain model. Two operations are comprised in editing an existing triangulation: it should be possible to add vertices and it should be possible to delete them. The latter is particularly important for filtering purposes, to reduce the multibeam data set to the most significant points.

Deleting points yields a re-triangulation of the star shaped polygon, which results when the vertex and its edges are eliminated. After deleting a vertex, the triangles containing this vertex become invalid, and a hole is created around this removed point. The edges of this hole define a new polygon. It can be demonstrated that it suffices to insert the conforming Delaunay triangulation of this polygon into the hole, in order to obtain the updated triangulation of the reduced dataset (Brouns *et al.*, 2001). By extension, when deleting a group of vertices, a big hole is created, which can be filled with triangles, and which can be reinserted in order to form the complete Delaunay triangulation of the reduced dataset.

Adding vertices one by one can easily be done by the incremental algorithm as it is the basic operation of this construction method.

3.3 Advantages and drawbacks of grid models versus TINs

It can be a requirement of the client of dredging / reclamation projects that the original measured points have to be included in the digital terrain model, from which the volumes are derived. This requirement allows the client to check the original results in a 3D modeling environment, which simplifies the manual appreciation of the point set. TINs are a favorite scheme to construct a DTM from a sea floor measured at discrete spots. Grid models, compared to TINs, have three important drawbacks:

1. It is generally impossible to have each sampled point of a TIN associated to one grid height/depth, since the measurements are not on a regular grid. Instead, the mutual distances depend on the survey ship's survey system (equally spaced measurements or not) and attitude (roll, pitch, yaw);

2. Grid values do not reflect the actual measurements, since gridding means either assigning interpolated values when the measurement density is inferior to the grid density, or resampling and loss of information, when the measurement density is superior to the grid size; the latter introduces unwanted "smoothing" in the DTM.

3. The grid model is not adaptive: whereas TINs will naturally represent areas with detailed relief information with a denser triangle pattern than areas with a smoother relief, grids will be far less flexible to cope with variable levels of detail.

TINs do not have these drawbacks, but they are more demanding towards computer memory and processing time Moreover, the algorithms needed for geometric computations are more sophisticated.

4. ACCURACY OF TIN BASED VOLUMES

4.1 Volume computation in a TIN

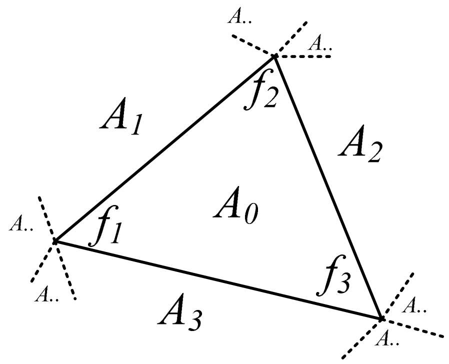
Volume computations in TINs or grid models are quite straightforward. In a TIN model, a prismatic volume is computed between a horizontal reference level and each triangle, on the condition that in planimetry, the triangle is inside the area where the volume has to be computed. Furthermore, it is assumed that the most important stochastic error is the altimetric error and that the planimetric positioning errors can be neglected.

Hereunder, the mathematical analysis of the different cases is performed.

4.2 TIN with irregular spaced points

With A_j as the planimetric surface of a triangle *j*, f_{ref} as the height in relation with the horizontal reference plane and f_i as the elevation of the 3 vertices *i* of the triangle, the volume V_j generated by one triangle *j* is equal to

$$(\frac{1}{3}(f_1 + f_2 + f_3) - f_{ref})\, A_j = V_j$$



The total volume is the sum of the volumes of all individual prisms, thus

$$\frac{1}{3} \sum_i f_i \left(\sum_{f_i \in \mathrm{A_j}} \mathrm{A_j}\right) - f_{ref}\, \mathrm{A_{tot}} = V$$

If we call B_i the sum of the surfaces of all triangles with point *i* as vertex or

$$B_i = \left(\sum_{f_i \in \mathrm{A_j}} \mathrm{A_j}\right)$$

Then we can write

$$\frac{1}{3} \sum_i B_i - f_{ref}\ \mathrm{A}_{\mathrm{tot}} = V$$

Assuming that all f_i are independent, the variance of the volume can be found

$$Var(V) = \frac{1}{9} \sum_i Var(f_i)\,\mathrm{B}_i^2$$

The standard deviation and variance $Var(f_i)$ of the elevation of a point is usually assumed to be constant ($f_i = f$) so that, with n the total number of points

$$Var(V) = \frac{Var(f)}{9} \sum_i \mathrm{B}_i^2$$

This form is useful in the case of a TIN model based on non-equidistant points.

The mathematical "minimal variance" solution is obtained when all surfaces B_i are equal, and as

$$\sum_i B_i = 3 \sum_i A_i$$

or

$$B_i = \frac{3A_{tot}}{n}$$

and

$$\sum_i B_i^2 = \sum_i \left(\frac{3A_{tot}}{n} \right)^2$$

We obtain

$$Var_{\min}(V) = \frac{Var(f_i)}{9} \sum_i \left(\frac{3A_{tot}}{n} \right)^2$$

or

$$Var_{\min}(V) = \frac{Var(f_i)}{n} A_{tot}^2$$

$$\sigma_{\min}(V) = \frac{\sigma(f_i)}{\sqrt{n}} A_{tot}$$

The mathematical theoretical "maximal variance" solution for the volume is when one surface B_i is maximal and all other B_i are neglectable and therefore set equal to zero. In this case the non-zero $B_i = 3A_{tot}$, and

$$Var_{\max}(V) = Var(f_i).A_{tot}^2$$
$$\sigma_{\max}(V) = \sigma(f_i).A_{tot}$$

Let's return to the standard case for which

$$Var(V) = \frac{Var(f_i)}{9} n\overline{\mathrm{B}^2}$$

$$Var(V) = \frac{Var(f_i)}{9}.n.((\overline{B})^2 + \sigma^2(B))$$

or

$$Var(V) = \frac{Var(f_i)}{9}.n.(\overline{B})^2.\left[1+\left(\frac{\sigma\ (B)}{\overline{B}}\right)^2\right]$$

and as

$$\overline{B} = \frac{\sum_i B_i}{n} = \frac{3\sum_j A_j}{n}$$

We get

$$Var(V) = \frac{Var(f_i)}{9}.\frac{1}{n}.(3\sum_j A_j)^2.\left[1+\left(\frac{\sigma\ (B)}{\overline{B}}\right)^2\right]$$

Finally

$$\sigma(V) = \frac{1}{\sqrt{n}}.\sigma(f).A_{tot}.\sqrt{1+\left(\frac{\sigma\ (B)}{\overline{B}}\right)^2}$$

The latter form is applicable to TIN's of irregular spaced points but is also particularly suited in the case of a TIN model based on equidistant points.

4.3 TIN with regular spaced points

Assuming a TIN of regular spaced points, and without the consideration of border issues, a minimum of the standard deviation can be found for a layout where all rectangular cells of the TIN have an identical direction of the diagonal. In this case, every non-border point has 6 neighboring triangles and as all triangles have the same surface, *σ(B) = 0*, it can be found that

$$\sigma(V) = \frac{1}{\sqrt{n}}.\sigma(f).A_{tot}$$

In case the diagonals in the grid system are alternating, half the number of non-border points have 4 neighbors and the other half 8 neighbors (Figure 2).

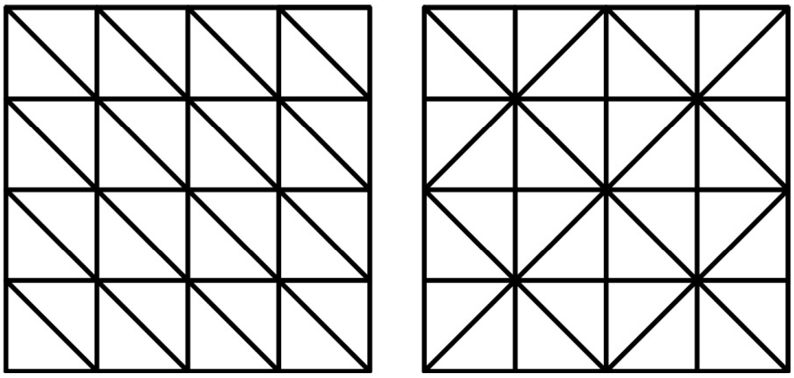


Figure 2: 6 neighbor triangles (left); 4 or 8 neighbor triangles (right)

Hence

$$\frac{\sigma\ (B)}{\overline{B}} = \frac{2}{6} = \frac{1}{3}$$

and

$$\sigma(V) = \frac{1}{\sqrt{n}}.\sigma(f).A_{tot}.\frac{\sqrt{10}}{3}$$

The difference in standard deviation of the volume between the optimal layout and the worst case layout is only a factor $\frac{\sqrt{10}}{3}$ or 5.4 % difference.

4.4 Border effects

As mentioned above, the mathematical analysis was made for non-border points in a TIN based on a equidistant points. The question arises if border issues give a significantly different result for the standard deviation of the volume in a TIN with equally spaced points.

In the following, we assume a geometric layout with 6 neighbors and a total of *n* points, *m* points are border points, meaning that they are laying along the edges of the triangulated zone, within the area where the volume is computed, and where the ration n/m is called φ, with 0 <= φ <= 1.

$$\overline{B} = \frac{6(n-m)+3m}{n} = 6-3\varphi$$

and

$$\overline{B^2} = \frac{36(n-m)+9m}{n} = 36-27\varphi$$

while

$$\left(\overline{B}\right)^2 = 36+9\varphi^2-36\varphi$$

thus

$$\sigma(B)^2 = \overline{B^2} - \left(\overline{B}\right)^2 = (36-27\varphi)-(36+9\varphi^2-36\varphi) = 9\varphi-9\varphi^2 = 9\varphi(1-\varphi) \approx 9\varphi$$

and

$$\frac{\sigma(B)^2}{\left(\overline{B}\right)^2} = \frac{9\varphi(1-\varphi)}{\left(6-3\varphi\right)^2} \approx \frac{\varphi}{4}$$

Finally

$$\sqrt{1+\frac{\sigma(B)^2}{\left(\overline{B}\right)^2}} = \sqrt{1+\frac{9\varphi(1-\varphi)}{\left(6-3\varphi\right)^2}} \approx \sqrt{1+\frac{\varphi}{4}} \approx 1+\frac{\varphi}{8}$$

As 0 <= φ <= 1, the maximal border effect on the standard deviation of the volume is an augmentation of the standard deviation with 1/8 or 12.5 %. Usually φ is close to 0 and the border effect on the standard deviation on the volume is neglectable. By extrapolation, this conclusion is also applicable to a TIN of a rectangular grid with different layout (e.g. 4/8 layout) or to a TIN with irregular spaced points.

4.5 Difference TIN model

The standard deviation of the volume difference between two TINs is given by taking the square root of the sum of the squares of the standard deviation of the volumes of both TIN's, thus

$$\sigma(\Delta V) = \sqrt{\sigma_1(V)^2 + \sigma_2(V)^2}$$

This formula can also be used in the case of a very dense (E,N) matrix interpolated in a TIN. Overlaying the sampled points of the TIN with a regular grid is a common technique used for volume computations between TIN's as the direct mathematical comparison of 2 TINs is a time-consuming computational operation. It is faster to generate a very dense (E,N) matrix of equidistant points and interpolate the height/depth values in each TIN model and subtract one height/depth from the other for each matrix point.

4.6 Example

The following small data set with 5 irregular spaced points given in (E, N, H) is considered:

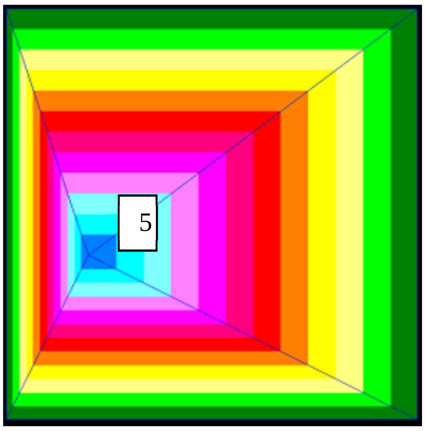
E	N	H
0	0	0
5	0	0
0	5	0
5	5	0
1	2	6

Table 1: Coordinates of the example

Besides, a standard deviation of 0.5 for each of the height values is given.
A Delaunay TIN is built with these 5 points and it yields 4 triangles. The different height zones have different colors (Figure 3).

The volume between a zero-level reference and this surface is easily computed as the volume of 4 prisms, with for each prism: 1/3 * planimetric surface * prism height. In this example, where the prism height is always equal to 6, the prism volume = 2*planimetric surface or for all 4 prisms: 2 * (0.5*5*1 + 0.5*5*3 + 0.5*5*4 + 0.5*5*2) = 2 * 2.5 * (1+4+2+3) = 50.

1 2



4 3

Figure 3: Elevation (contour) model of the exemplary points

The variance of the volume is $\mathrm{Var}(V) = \frac{1}{9}\sum \mathrm{Var}(f_i) B_i^2$. *$Var(f_i)$* is computed as the square of the standard deviation of the heights or 0.5^2=0.25. The five different B_i for point 1, 2… 5 are

B_1 = 0.5*5*1+0.5*5*3	= 10
B_2 = 0.5*5*3+0.5*5*4	= 17.5
B_3 = 0.5*5*4+0.5*5*2	= 15
B_4 = 0.5*5*2+0.5*5*1	= 7.5
B_5 = 0.5*5*1+0.5*5*3+0.5*5*4+0.5*5*2	= 25

As a check, the sum of the B_i is always 3 times the total surface (75 = 25*3).
The sum of the B_i^2 is 100 + 306.25 + 225 + 56.25 + 625 = 1312.5
Hence the Var(V) is 0.25 * 1312.5 / 9 = 36.458
The standard deviation for the volume of 50 is the root of 36.458 or approximately 6.038.
Thus, the volume between the zero-level and the prism surface is 50 +/- 6.038.

If point 5 had been the central point with coordinates (2.5, 2.5, 6), the volume would have been the same. B_1, B_2, B_3 and B_4 would be all equal to 0.5*5*2.5+0.5*5*2.5=12.5, B_5 being equal to 25. Var(V) is then 0.25 * ($4*12.5^2+25^2$) / 9 = 34.722 and the standard deviation is 5.893.

The theoretical "lower limit" case would yield $\sigma_{min}(V)$ = sqrt($0.25*25^2$ /5) = 5.590, and the "upper limit case" would yield $\sigma_{max}(V)$ = 12.5. However, these are theoretical cases, both layouts being geometrically impossible.

5. CONCLUSIONS

Multibeam echosounder data impose some specific requirements to the processing. These requirements have been identified and the different aspects of DTM construction by grid modeling and by Delaunay triangulation have been treated in this context and opposed to each other as two alternatives of which the advantages and drawbacks have been discussed.

Editing the model is significantly more complex when TINs are used in comparison to regular spaced grids. As an example, merging two overlapping triangulation sets was demonstrated. The authors use an adapted merge-step in the divide-and-conquer algorithm to replace old data in an existing triangulation by newly available data. TINs are to be preferred when the surveyed area has a non-homogeneous coverage.

Equidistant grid models are less flexible, but offer higher speed, lower memory and easier implementation algorithms as most important assets, making them to be preferred when the measured area is homogeneously covered by a high-density multibeam survey. For heterogeneous covered areas, typical for single beam surveys, TINs are a priori the preferred option. A mathematical form for the standard deviation of TIN based volume computations is proposed, either based on irregular spaced points or not, with derivation of the best and worst case solutions. These computations are illustrated in an easy-to-follow example. It was demonstrated that border effects does not have a significant influence on the standard deviation of volumes in a TIN. In the near future tests with full-scale samples of a few million points will be performed and reported.

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BIOGRAPHICAL NOTES

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