

# The Optimum Procedures of Determining the Coefficient of Linear Thermal Expansion and Calibration of Precise Leveling Rods

Mariusz FRUKACZ, Poland

**Key words:** Invar, coefficient of thermal expansion, calibration, precise leveling staff

## SUMMARY

The author proposed the procedures of the laboratory testing of precise leveling rods in the Geodesy Metrology Laboratory at the AGH University of Science and Technology in Krakow (GLM AGH), connected first of all with marking the coefficient of the linear thermal expansion (CLTE) of the Invar band and investigation the scale of graduation of leveling rods. The author designed and executed the modernization of the thermal chamber. Analyzing thermal properties of Invar author made series of investigations having the aim of the optimization of procedures of marking CLTE and ways of calculating the thermal correction to the results of the precise leveling. The author showed new factors which have the essential influence on the value of CLTE like the phenomenon of the thermal anomalies of Invar and temperature hysteresis.

The author analyzed the construction and functions of vertical comparator, results of 165 independent investigations conducted with his utilization and characterized the procedure of calibrating the precise leveling rods. The author gave the most essential problems appearing executing the calibration, described the principal differences of the individual types of rods and described changes of scale of the graduation resulting from the outflow of time and the inappropriate exploitation of leveling rods. The author also proposed the modification of the algorithms of calculating the calibration correction to the results of the precise leveling.

# **The Optimum Procedures of Determining the Coefficient of Linear Thermal Expansion and Calibration of Precise Leveling Rods**

**Mariusz FRUKACZ, Poland**

## **1. PREFACE**

To measure the basic leveling network, the technique of precise leveling is generally applied, and the height of points is determined in the required scale, given by the graduation put on the Invar band of the rods. Exact definition of actual length (scale) of the graduation of rods makes one of basic tasks of surveying metrology. In the process of calibration a good model can be a master or a relevant measurement system designed to implement or reproduce the measurement unit. Such a system using a laser interferometer, in the best way reproduces the required measurement unit, which increases the accuracy of calibration and allows the reproduction of barcode leveling rods, in the relation to which classical procedures cannot be applied. The computer-controlled interferometer allows to increase the degree of the automation of measurements, due to which the influence of personal errors is minimized and the efficiency of the reproduction process is enhanced. However, to fully use the possibilities brought by the application of laser interferometer, one should implement the adequate technology of measurements. It is necessary to construct proper measurement stands, which provide high accuracy of the measurements. The specifics of laser interferometry also requires making a relevant way of monitoring and taking into account in the calculations the values of physical conditions, which is possible only in the adjusted to this purpose metrological laboratory. One should also propose research procedures, which in a maximal way could be adjusted to the possibilities of the interferometer, and at the same time will allow obtaining such data, which would describe the calibrated equipment in the best possible way.

The article presents the procedures of laboratory studies of precise leveling rods, applied in Geodesy Metrology Laboratory in AGH University of Science and Technology in Krakow (GLM AGH), in particular connected with the determination of the coefficient of linear thermal expansion (CLTE) of the Invar band and the studies on the scale of graduation of leveling rods in the vertical comparator, equipped with laser interferometer. The functioning check points, calculation procedures and the most important conclusions were presented.

## **2. ANALYSIS OF METHODS FOR TESTING LEVELING RODS IN LEADING RESEARCH CENTRES**

At present in Europe there are only a few centers dealing with marking the coefficient of linear thermal expansion of the Invar band and the calibration of leveling rods. The most modern solutions are applied in the Technische Universität München [Maurer 2000], Technische Universität Graz [Woschitz 2003] and Finnish Geodetic Institute [Takalo, Rouhiainen 2004]. Applied there solutions vary in construction and the very idea of calibration, however the optimal solution is the calibration leveling rod in the vertical position, thus the same position as the rod takes during the surveying. There is no clear

answer to the question if the calibration should be done to the graduation of rods or to the whole system leveler/rod [Hennes, Ingensand 2000]. The first solution has been applied for decades, and brings full information on the graduation of the leveling rod – both classical and barcode graduation. In the second solution, adequate only to barcode leveling rods, the summary scale of the set is determined, which consists of the graduation scale of leveling rods, charge-coupled device (CCD) applied in the digital leveler and influence of the calculation algorithm. This solution has many advantages (ergonomics and economy of the measurement, calibration of all the components at the same time), but there are also doubts about the meaning of such a concept. Firstly, the influence of the CCD of the leveler in the scale of the whole system is small, on average 1.2 ppm [Woschitz 2005]. Secondly, the calibration of the set is generally done for one or maximally a few target lengths – while in practice the range of target lengths is changing on each check point. Atmospheric conditions are also monitored and the light conditions for the leveling rod in the laboratory are different than in the field. According to the author, the basic way of the calibration should be the calibration of the graduation and its supplementation could be the calibration of the whole set leveler/rod.

### **3. DETERMINING THE COEFFICIENT OF LINEAR THERMAL EXPANSION (CLTE) OF PRECISE LEVELING ROD'S INVAR BAND**

#### **3.1 Thermal chamber for determining CLTE at GLM AGH**

The determination of CLTE at GLM AGH has been carried out since 1999. In 2003 – 2005 there was a modernization of this stand. In the framework of this modernization the thermal chamber was re-designed and a new system of the regulation and registration of system's temperature was proposed (Fig. 1).



Fig. 1. General view of the thermal chamber

As a result of the carried out verification and calibration procedures in the whole system, the mean error of determining the band temperature was determined on the level  $\pm 0.05^{\circ}\text{C}$ , and the mean error of length changes was  $0.8\ \mu\text{m}$ . The temperatures obtained in the chamber range from  $-10^{\circ}\text{C}$  to  $+50^{\circ}\text{C}$  and this, together with the laboratory in Munich, is the highest range of temperatures, obtained in thermal chambers to examine the leveling rods. The observation cycle to determine the CLTE for one leveling rod is 12-15 hours in a standard procedure 9 thermal thresholds are applied in the following order:  $20^{\circ}\text{C}$ ,  $10^{\circ}\text{C}$ ,  $0^{\circ}\text{C}$ ,  $10^{\circ}\text{C}$ ,  $20^{\circ}\text{C}$ ,  $30^{\circ}\text{C}$ ,  $40^{\circ}\text{C}$ ,  $30^{\circ}\text{C}$ ,  $20^{\circ}\text{C}$ . The values observed for every determined thermal threshold are the temperature of the Invar band of the leveling rod and changes in the length of the range of measurements (measured with the application of spiral microscopes). The obtained results are collected in the author-made program, which automatically makes necessary calculations and graphical presentation of the results. The method of least squares is used to calculate CLTE, which is then taken into account during the determination of thermal corrections to the measured values. The assessment of the accuracy and quality of the adjustment of the model is made. The mean error in determining the CLTE is on the level of  $0.01\div 0.05\ \text{ppm}/^{\circ}\text{C}$  for new barcode leveling rods,  $0.02\div 0.10\ \text{ppm}/^{\circ}\text{C}$  for barcode leveling rods already used in the field and  $0.07\div 0.15\ \text{ppm}/^{\circ}\text{C}$  for classical rods in wooden lining. This uncertainty is comparable with the obtained in highly reputed research centers in the world. After determining the sought parameters with the proper analysis accuracy, the graph presenting the relationships between relative length changes and temperature changes of the Invar band of the leveling rod is generated automatically and the certificate of CLTE determination is generated.

### 3.2 Thermal anomalies of Invar

The author's research showed that the determined value of the coefficient is significantly influenced by:

- the rate of temperature changes during the process of determining the coefficient,
- the choice of the moments of the observations,
- temperature stability of the Invar band.

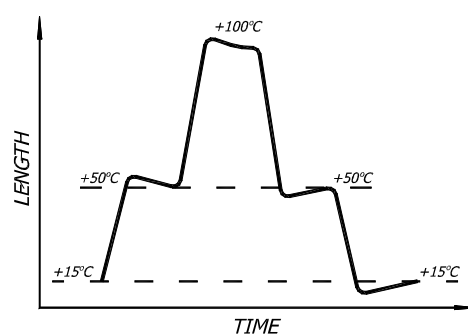


Fig. 2. Thermal anomalies of Invar

These factors cause the phenomenon of the anomaly of the changes in the Invar length, meaning much greater elongation of the greater increase of length differences during rapid heating than it would be at very slow changes. After the stabilization of the temperature length difference is getting smaller until reaching the length resulting from function relation (illustrated in Fig. 2). Already at the beginning of 20<sup>th</sup> century a famous French Nobel Prize winner Charles Edouard Guillaume (1861-1938) noticed the phenomenon of anomaly [Guillaume 1920], but this phenomenon has not been described in detail so far in geodesy literature. Anomalies of the changes of the Invar length can cause errors in the determination of the values of the coefficient (up to 0.5 ppm/°C) and consequently deformation of the results of the measurements, which mainly depends on the value of the measured length difference and the temperature of the measurement. The greater de-leveling of the area and difference between temperature of measurement and temperature of calibration, the greater the error (it will be 0.3 mm for the section of length difference 30 m leveled in temperature of 0°C). To eliminate the influence of the thermal anomaly of Invar the manner of the insolation of the system was changed and the procedure of determining the values of CLTE was modified, by the introduction of strict requirements referring the time of the temperature stabilization in the chamber.

### 3.3 Temperature hysteresis during determination of CLTE

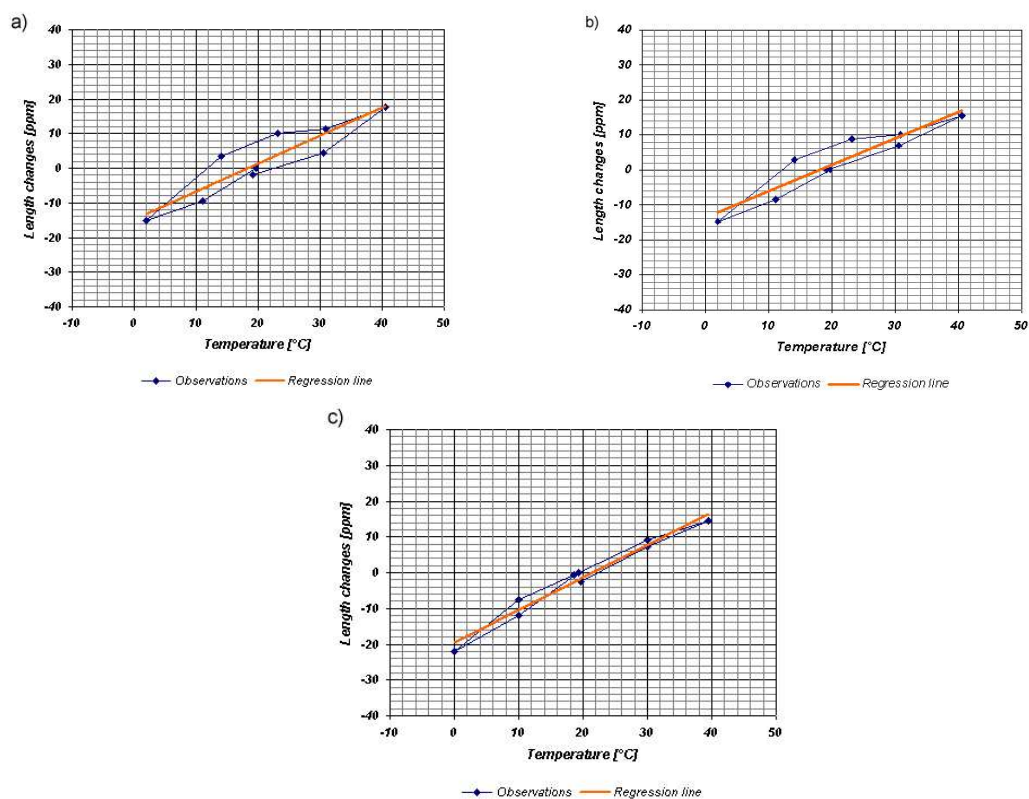


Fig. 3. Temperature hysteresis for Zeiss 13405 invar rod: (a) measurement in 2000, (b) measurement in 2000 with Guillaume correction, (c) measurement in 2007.

Analyzing the graphs of determining CLTE in many cases it can be seen that the course of length changes is characteristic for temperature hysteresis (Fig. 3a). One of the ways to minimize this phenomenon is introducing a so-called Guillaume's correction, which when was introduced to the results before the modernization of the chamber, improved the adjustment of simple regression by 80% (Fig. 3b). To determine CLTE, after the modernization the introduction of this correction is not justified, because new construction and research procedure significantly eliminated the occurrence of temperature hysteresis (Fig. 3c).

### 3.4 The phenomenon of nonlinear changes in the length of Invar

Accepted in surveying way of introducing thermal corrections assumes linear character of changes, which often does not find confirmation during the determination of CLTE of the rods' Invar band to precise leveling (Fig. 4).

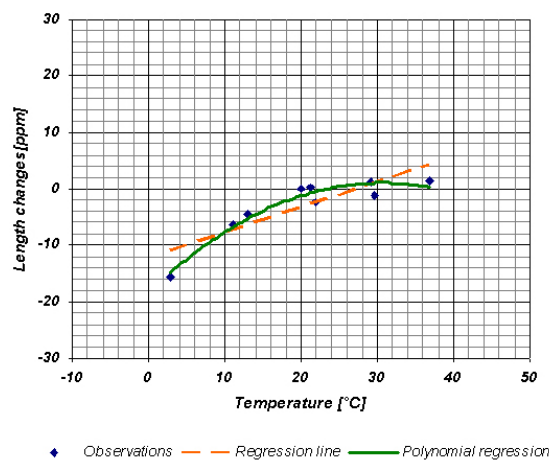


Fig. 4. Nonlinear changes in the length of Invar (rod: Topcon 16408)

The author's program to calculate CLTE determines three models for each case (linear regression and polynomial regression of second and third degree), which are then analyzed by the user. From the set of 50 CLTE markings for 10% rods it was required to apply the polynomial of second degree, while for none of the rods the reason for applying polynomial of third degree was found. Thus it can be accepted that the model of the polynomial of second degree is sufficient to describe non-linear changes in the length. Taking into account the non-linear character of length changes in the proposed by the author way of calculating the thermal correction caused the differences in the value of this correction reaching 0.5 mm. It seems that it is right to regard the phenomenon during the formation of the equations of thermal corrections.

## 4. CALIBRATION OF PRECISE LEVELING RODS IN GLM AGH USING LASER INTERFEROMETER

### 4.1 Vertical comparator - the station for calibration of leveling rods

The studies of leveling rods in GLM AGH is carried out on the vertical comparator. Its first concept was created at the Department of Surveying and Cartography AGH in 1998 – 1999. Modernization carried out in 2004 allowed for example introducing the measurement with the application of CCD camera [Beluch et al. 2008]. As a result of works also the two-axis measurement was introduced, which makes it possible to study the graduation of the leveling rod and determine the shape of the rail (Fig. 5). The application of this system contributed to the minimization of systematic errors resulting from the fact that the rail was not straight.

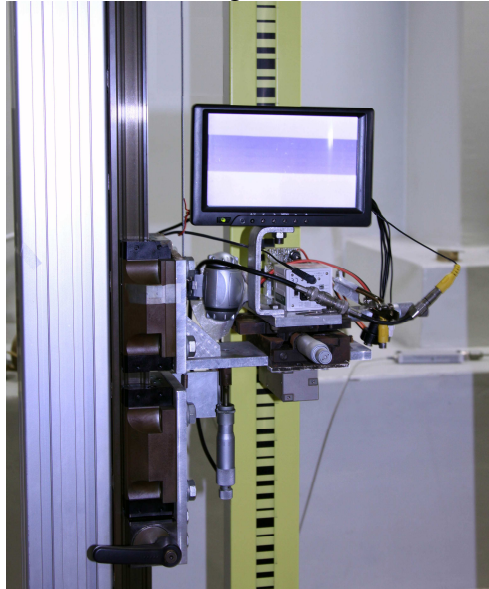


Fig. 5. View of a part of the vertical comparator

#### 4.2. The procedure of the calibration of leveling rods on the vertical comparator

To the main tasks of the multi-stage procedure of the calibration of leveling rods is the determination of zero of the graduation of the leveling rod and marking the scale and errors of graduation. Records during the calibration is carried out in the established intervals of the distance from the foot of the leveling rod, depending on the applied graduation and called „measurement points”, the number of which ranges from 60 to 75. For each point the records of the interferometer are reduced by the influence of atmosphere on the course of the laser beam, the change of the length of graduation is caused by thermal influences and the correction of the measurement system. The following stage is calculation of the corrections of graduation  $r_D$ , which are differences between the nominal situation of the graduation mark  $D$  and its situation determined during calibration  $L$ . Standard deviation of these corrections can be interpreted as the mean value of the error of the marking of the graduation marks on the Invar band of the leveling rod. The solution of the equation system by the least squares method for corrections  $r_D$  allows the calculation of the scale and the zero of the graduation of the leveling rod and make the assessment of the model. The final effect of the leveling rod calibration is its qualification for making the measurements, which is used in surveying to introduce the corrections the results of leveling. The formal document containing all this

information is „Certificate of Calibration”.

## 4.2 Analysis of research results

### 4.2.1 Research material

Based on the results of 165 calibration rods carried out w GLM AGH from December 2000 to December 2009, the author carried out many detail analyses, the results of which made basis proposal of further modernization and changes of the algorithm of calculations. From the carried out analyses it seems that after the modernization of the comparator and modification of research procedures, the zero of the graduation of the leveling rod is determined with the standard uncertainty on the level of  $\pm 5 \mu\text{m}$ , and the uncertainty of determining the scale of the leveling rod is about  $\pm 2 \text{ ppm}$ . Corrections of the scale for classical rods are in the graduation  $\pm 30 \text{ ppm}$ , and the mean value of the corrections of the scale for these rods is  $+7,1 \text{ ppm}$ . For the rods with the barcode this interval is much narrower and it could be accepted that, it is  $\pm 15 \text{ ppm}$  (Fig. 6).

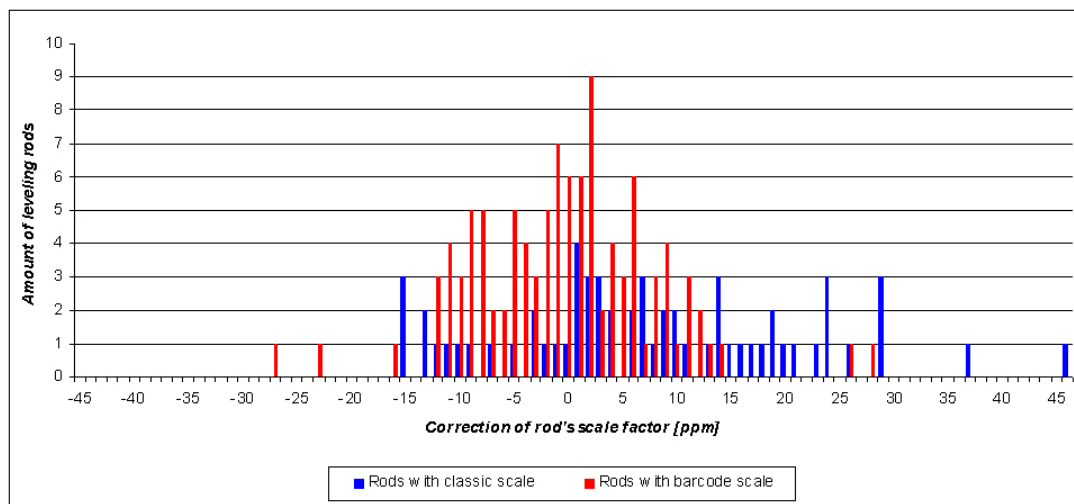


Fig. 6. Amount of leveling rods with specified correction of scale factor

### 4.2.2 The comparison of calibrations made in different periods

Mentioned earlier made several times calibrations of the same rods allow following the changes of their metrological parameters, as well as the consequences of subsequent modernization of the comparator. Analyzing many factors (personal errors, changes in methodology, changes of meteorological conditions during making the measurement) it was found that there was the influence on the results of calibration on the prototype comparator (before the modernization) had the temperature in GLM during measurement. Fig. 7 presents the mean changes of the scale of the graduation of the rods for each period and mean changes of temperature between calibrations (to increase the legibility the changes of temperature were multiplied by ten).



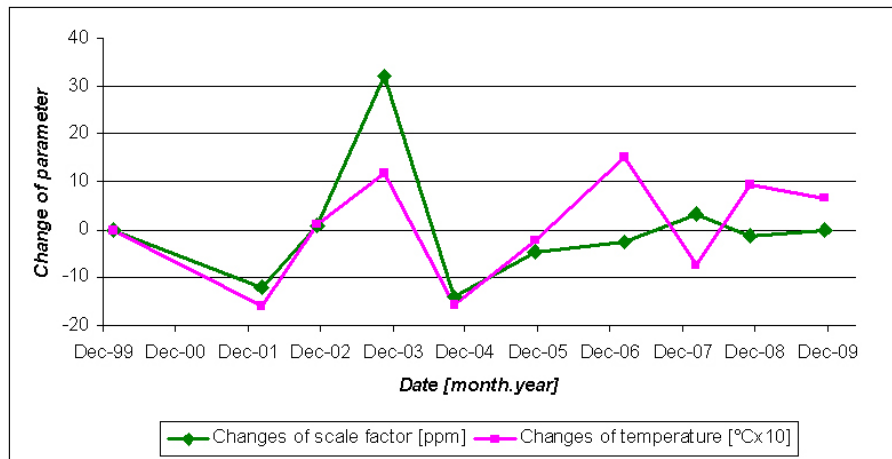


Fig. 7. Changes of rod's scale factor and average temperature between calibrations

This graph confirms the relationship between changes and the scale of temperature for the prototype comparator, while for the comparator after the modernization the changes of the scale of graduation (not exceeding  $\pm 3$  ppm) could be regarded independent from the changes of temperature. This results from the differences in the construction of both comparators (the column of the comparator that was made of aluminum was more susceptible on thermal influence), as well as changes in the technology of measurement (separate measurement of the rail curvature and graduation were replaced with joint measurements). This way the modernization of the comparator contributed to the increase of thermal stability of the construction and allowed definitely better determination and minimization of systematic errors.

#### 4.2.3 Changes of the zero value of the leveling rod graduation

It is also interesting to compare the changes of the zero value in Fig. 8. In the graph it can also be seen that the zero of the graduation is systematically increasing or decreasing for the whole groups of rods calibrated in the same time. Similarly like in the case of the changes of scale, also here the changes are mainly visible for the measurements made on the prototype comparator. However systematic decrease of the zero of the graduation can also be seen for the last measurement (November-December 2009) for all the calibrated rods. In many cases the differences between subsequent determinations of the zero of graduation exceeds the value of the limit error to define zero of the graduation (equal  $\pm 30 \mu\text{m}$ ). The factor influencing the results in such a great degree was personal influence, in particular the interpretation of the place where the node mark touches the foot of the leveling rod. This identification was not unambiguous – depended on the direction of light, the state of the foot of the leveling rod, personal features of the observer – and zero value of the graduation and could change even by  $40 \mu\text{m}$ .

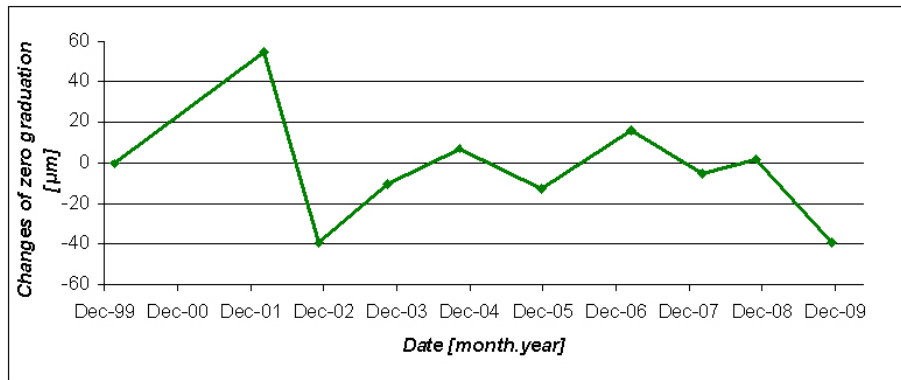


Fig. 8. Changes of the zero value of graduation between calibrations

Due to the detection of personal influence a new technology of determining zero of graduation, in which the optic observations were replaced with the image of the intermediate element (the fragment of the code), put in the permanent, precisely determined distance from the spherical surface of the node mark. Nowadays the repeatability of determining zero of the graduation of the leveling rod is  $5 \mu\text{m}$ .

#### 4.2.4 Abnormal changes of graduation

The parameter of the scale of the division of the leveling rod assumes a linear character of the mainly in such cases the earlier speculations could be applied, however the graduations of many rods do not have a linear character (Fig. 9). Often the cause of such a situation is significant deformation of the aluminum body of the leveling rod, caused by fixing the leveling supports in the half of its length.

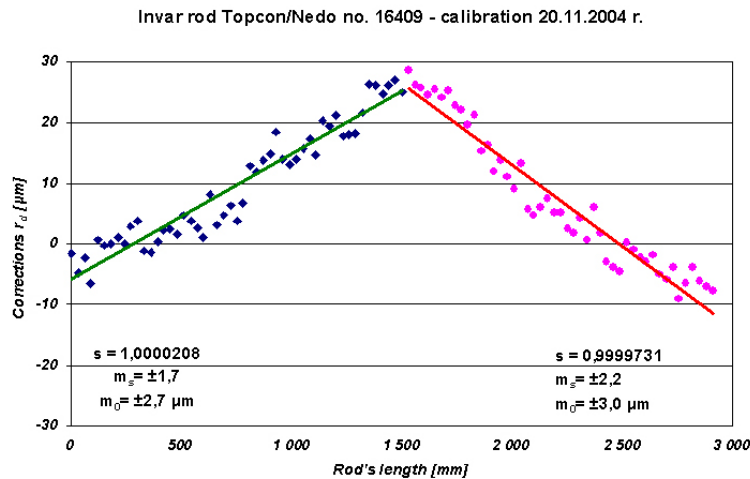


Fig. 9. Two different scale factors for graduation of Invar rod no. 16409

In the case of the deformed in such as way rods we deal with the rapid change of scale in the place of flexion of the profile. Treating both fragments of graduation separately and carrying out for them separate approximations, we obtain two scales of graduation, varying not only in

the value, but also the character of changes: before the deformation this graduation is elongated compared to the nominal, while after the damage the graduation shrinks. The difference between the scales of both parts of graduation is even 50 ppm. Such a deformation of the graduation of both rods can cause the appearance, especially during the field measurements in the area of constant inclination, of systematic errors of the determined length difference. Assuming the length difference of about 1 m on the station and the measurement with short targets, the records from two rods will be done on the fragments of rods of different characteristic of graduation, which can cause changes in the height difference of 20 – 30  $\mu\text{m}$  per station. For the leveling of 20 such stations the error will be 0.4 – 0.6 mm.

#### 4.2.5 Nonlinear changes in the graduation of leveling rod

Analyzing the graphs of the corrections of graduation  $r_d$  of the calibrated rods one can notice that in many cases the approximation with the straight line does not reflect their real system. Such a situation usually occurs in the case of rods of deformed body (e.g. cases described above), but also for many rods with classical graduation, which was brought on Invar band with the method of print screen [Fisher, Fisher 1999]. Such a technology of forming graduation was subdued to many factors changing the nominal system of dashes: starting from lower precision of making, using the pattern of graduation and repetitive errors. [Maurer 2000]. In such situations in the calculation process additional models of polynomial regression were taken into account. Fig. 10 presents the difference in approximation of linear regression and polynomial regression of fifth degree.

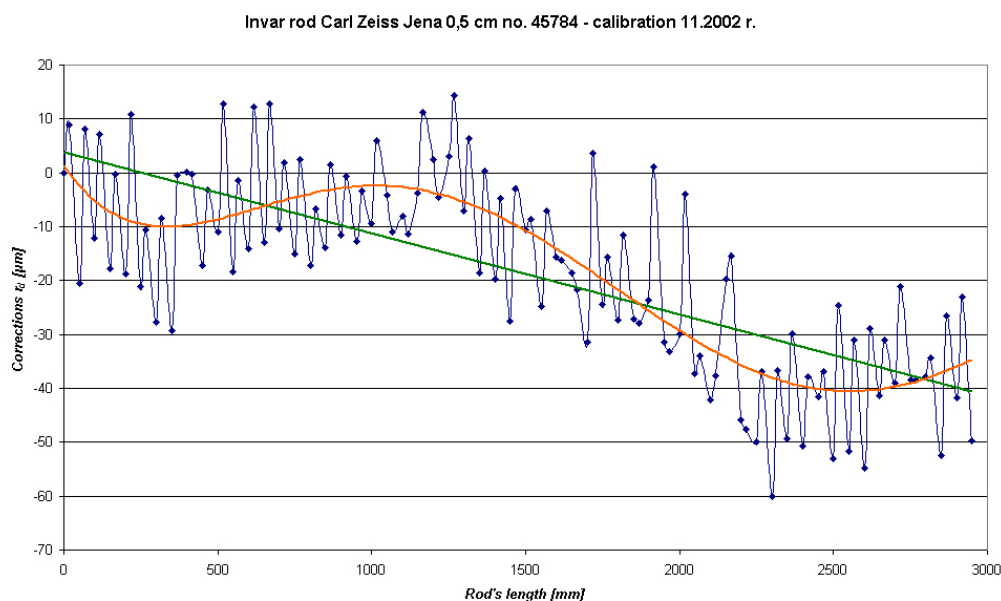


Fig. 10. Nonlinear changes in the graduation of leveling rod

In the cases of such rods, during calculating the calibration correction to the results of the leveling, it is more justified to supplement each record of the leveling rod with the determined

for this deviation  $r_d$  or to apply the approximation of the deviation of graduation marks for the nominal values with a relevant polynomial [Frukacz 2005].

## 5. CONCLUSIONS

Based on the carried out analyses of literature and studies carried out in the Geodesy Metrological Laboratory of AGH-UST it was stated that the optimization of the procedures of determination of CLTE and calibration of rods allow the diminishing of systematic errors in precise leveling. Applied so far procedures of introduction calibration and thermal corrections to the results of leveling could cause the appearance of errors, the value of which was higher than the influence of other factors minimized by the corrections (e.g. the phenomenon of refraction).

Detail studies on the determination of the coefficient of linear thermal expansion of the Invar band of the leveling rod confirmed the meaning of the modernization of thermal chamber. Owing to the modernization of the chamber and detail procedures of measurement the mean error of CLTE was decreased by 0.02 ppm/°C. For new rods unused in the field, this error was  $\pm(0.01 - 0.03)$  ppm/°C, which is comparable to the values obtained in TU München ( $\pm 0.02$  ppm/°C). Proposed by the author solutions of construction and measurement eliminated or significantly minimized the neglected earlier in metrology of rods influence of thermal anomaly of Invar and temperature hysteresis on the obtained results. Software made for this research also allows efficient diagnoses and describes of non-linear changes in the length of Invar band. In 10% of cases the necessity of the application of polynomial approximation of second degree was found, which enabled the author to propose the modified way of calculating thermal correction to precise leveling. Comparison of the obtained results with the results of other highly reputed centers in Europe confirms that the accepted solutions in precise leveling are right and the accuracy of determining of CLTE is high.

Analyzing the results of calibration one should state that the reached parameters of accuracy, as well as the obtained results of calibration do not vary from the ones in the most highly reputed metrological laboratories in the world. The mean uncertainty of determining the scale of the graduation of classical rods in GLM AGH is now  $\pm 2.5$  ppm, and the rods with the barcode graduation  $\pm 2.1$  ppm (to compare: in TUM this value is  $\pm 2$  ppm, and in TUG the scale of the set barcode rod/digital leveler is defined with the uncertainty of  $\pm 3$  ppm). The carried out modernization of the comparator, as well as the range of introduced modifications of the calculation algorithm allow the examination of errors in graduation with the uncertainty by 1 ppm smaller than the one of the prototype comparator, and increased accuracy of measurements also allows the analysis of the factors earlier difficult to detect, such as the influence of the way of using the leveling rod on its graduation and changes of the value of scale in time. The proposed ways of calculating calibration correction, especially in the cases of leveling requiring enormously high accuracy and the measurements of dislocations of high hydro-technical objects, decrease the influence of graduation errors on the final results of leveling.

## REFERENCES

- BELUCH J., FRUKACZ M., MRÓZ J., POKRZYWA A., SZCZUTKO T. (2005) *Interpretation of the Results of Calibration of High Precision Leveling Staffs and Remarks Arising out from Leveling Staffs Testing*, Reports on Geodesy, nr 3, s. 103–114, Warszawa.
- BELUCH J., FRUKACZ M., MRÓZ J., POKRZYWA A., SZCZUTKO T. (2008) *Badania laboratoryjne niwelatorów i precyzyjnych łąt niwelacyjnych*, Kraków.
- FISCHER T., FISCHER W. (1999) *Manufacturing of High Precision Leveling Rods*, [w:] Lilje M. [red.] *The Importance of Heights*, FIG, Gävle.
- FRUKACZ M. (2005) *Badania nad wyznaczeniem i wprowadzaniem poprawki termicznej oraz kalibracyjnej do wyników niwelacji precyzyjnej*. Technical Sciences, Supplement 2, Publisher UWM Olsztyn, s. 283-292.
- GUILLAUME Ch.-E. (1920) *Invar and elinvar*. Nobel Lecture, 11.12.1920.
- HENNES M., INGENSAND H. (2000) *Komponentenkalibrierung versus Systemkalibrierung*. [w:] Schnädelbach K., Schilcher M. [red.] *Ingenieurvermessung 2000*, Wittwer Verlag, Stuttgart.
- MAURER W. (2000) *Kalibrierung von Präzisions-Nivellierlatten*. Ingenieurvermessung, München.
- TAKALO M., ROUHIAINEN P. (2004) *On System Calibration of Digital Level*. 14<sup>th</sup> International Conference on Engineering Surveying, Zürich.
- WOSCHITZ H. (2003) *System Calibration of Digital Levels: Calibration Facility, Procedures and Results*. (Dissertation Technische Universität Graz) Shaker Verlag, Aachen.
- WOSCHITZ H. (2005) *Systemkalibrierung: Effekte von digitalen Nivelliersystemem*. AVN Allgemeine Vermessungs Nachrichten 6, s. 239-244.

## BIOGRAPHICAL NOTE

**Mariusz Frukacz** – Doctor of Technical Sciences. Studied surveying at the AGH University of Science and Technology in Krakow (1995–2000). He finished his doctoral studies in 2010. He's interested in geodetic metrology and precise leveling. Publications in various journals and conference proceedings. He's also graduated from film and media studies at Jagiellonian University in Krakow, Poland.

## CONTACT

PhD Mariusz Frukacz  
AGH University of Science and Technology  
Faculty of Mining Surveying and Environmental Engineering  
Department of Geomatics  
al. Mickiewicza 30  
31-315 Krakow  
POLAND  
Tel +48 12 617 38 92  
Fax +48 12 617 45 88  
Email: frukacz@agh.edu.pl