Empirical Models of Vertical Crustal Motion in the Great Lakes Region

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Key words: GRACE, robust least-squares, vertical crustal motion, vertical datum

SUMMARY

The new geoid-based vertical datum in Canada CGVD2013 provides the physical height component of a 3-dimensional spatial reference frame and enables a direct transformation of physical heights to ITRF. Models of the geoid rate of change and vertical crustal motion should also be supplied with the vertical datum for correcting various height data sets for different observation epochs, thus ensuring a temporal data homogeneity in areas with secular geodynamic processes. One of these areas is the Great Lakes region experiencing a significant tilt due to the glacial isostatic adjustment of the crust. The models of vertical crustal motion and geoid rate can come from combining local, geometric geodetic observations from GPS/GNSS with global temporal gravity data from GRACE. Particular challenges include minimizing the hydrological signals leakage over the area of interest that contaminates the GRACE-derived rates due to the geodynamic process in the region, as well as deriving point estimates from spatially integrated gravity data.

In this study, we combine GRACE-observed rates of gravity change converted to vertical crustal motion and GPS/GNSS vertical velocity data in the Great Lakes region. The combined vertical motion model is realized via an interative least-squares adjustment procedure including variance-component estimation and robust outlier detection.

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1. INTRODUCTION

A new reference system for orthometric (physical) heights was adopted in Canada in November 2013. This state-of-the-art vertical datum, which integrates most recent GOCE global gravity field models, regional gravimetry and altimetry data as well as high resolution digital elevation data, was developed and is maintained by the Canadian Geodetic Survey, Natural Resources of Canada (Véronneau and Héroux, 2006; Huang and Véronneau, 2013). In cooperation with the National Geodetic Survey, NOAA, USA, a new continental geoid-based vertical datum is in preparation and will likely be adopted by 2020 (NGS, 2008). Rates of change of the vertical datum are in demand for precise engineering and geodetic surveys, water level studies in the lakes, as well as glacial isostatic adjustment studies. Models of the rates of the vertical datum have been developed in the Department of Geomatics Engineering at the University of Calgary since the early releases of the GRACE gravity mission data (Tapley et al., 2004). Rangelova et al. (2009a) showed that if one-centimetre accuracy of the geoid-based vertical datum in Canada is targeted, the geoid rate needs to be taken into account every decade while the vertical crustal motion appears to be much more important and should be taken into consideration after 2-year intervals.

The Great Lakes region constitutes a really important domain for vertical crustal motion studies with a high concentration of permanent and campaign GPS/GNSS observations. Our study focuses on the optimal combination of the available GRACE and GPS/GNSS data presented in section 2 for developing empirical crustal motion models in this region. The optimal combination procedures described in section 3 are based on the iterative re-weighting and the traditional least-squares adjustment with subsequent variance-component estimation. The developed models are compared and discussed in section 4 together with the estimated a-posteriori data errors.

2. DATA

2.1 GPS/GNSS vertical crustal velocities

A set GPS/GNSS vertical velocities in the Great Lakes region (Figure 1) is available from published velocities derived from permanent and repeated observations in the Canadian Base Network (e.g., Sella et al., 2007). The vertical velocities show a large northeast/southwest slope. The large uplift of 7-8 mm/yr in the northeast areas sharply decreases to zero in the lakes area, where the GPS/GNSS observations place the line of zero motion. This is an important model constraint in glacial isostatic adjustment studies. Only a diagonal error variance-covariance matrix is available for this data set.

2.2 GRACE-derived vertical motion rates

The GRACE vertical motion rates are derived from 144 CSR RL05 monthly solutions. This data set covers the time period from April 2002 until August 2015. A mean gravity field is calculated up to the spherical harmonic degree and order 96 by averaging all of the available monthly solutions. Changes from the mean are obtained by subtracting the mean field from each monthly solution. 17 missing monthly solutions are predicted by interpolation taking into account the mean, trend, annual and semi-annual signals in the available data. The typical post-processing GRACE data procedure includes filtering of the correlated errors in the spherical harmonic coefficients followed by smoothing of the random and residual correlated errors. Vertical motion rates are calculated by an approximate formula by Wahr et al. (2000) and are given in Figure 1 for the Great Lakes region. The superposition of the two data sets in the figure shows clearly that the GRACE vertical motion surface has a steeper gradient than the GPS/GNSS vertical velocities. Moreover, the line of zero motion is displaced to the south of the lakes. Similar to GPS/GNSS, the GRACE-derived glacial isostatic signal peaks in the northeast areas, but the maximum rates are about 2 mm/yr higher than the maximum GPS/GNSS vertical velocities. To balance the two data sets, the GRACE-derived vertical motion rates are interpolated at the GPS/GNSS station locations using the multiquadrics explained in section 3.

The two data sets have very different errors. The errors of the GRACE-derived rates are associated mainly with hydrological long-wavelength signals leaked over the area of study. Although these are removed from the GRACE data in the data post-processing stage, residual hydrological signal still exists and may show up as long-wavelength pattern due to the smoothing applied to the GRACE data. Other potential error sources are the different reference epochs of the data sets, rates in degree one harmonics not present in the GRACE gravity solutions, local monument instabilities (assumed to be largely filtered out in the original GPS/GNSS data adjustments), as well as local crustal uplift/subsidence not associated with the glacial isostatic adjustment in the region.

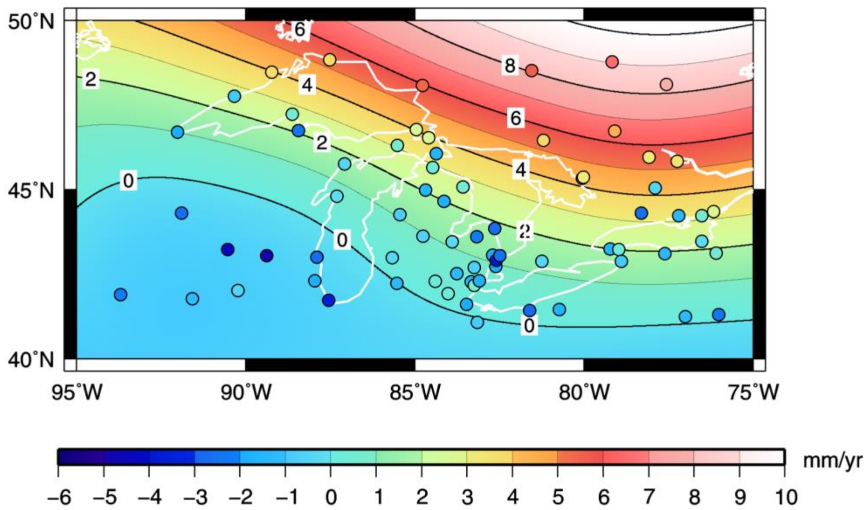

Figure 1. CSR RL05 vertical motion rates and GPS/GNSS vertical velocities.

A diagonal error variance-covariance matrix is used in the least-squares adjustment and variance-component estimation. This matrix is computed by orthogonalization of the fully-populated error variance-covariance matrix of the vertical motion rates derived by error propagation of the calibrated standard errors of the GRACE monthly gravity solutions.

3. METHOD

An optimal combination procedure based on the iterative re-weighting least-squares (IRLS) method is designed to include

1. identifying outliers and minimizing their effect on the combined velocity surface, and
2. estimating the relative errors of the data sets using the BIQUE variance component estimation (VCE, Rangelova et al., 2009b) .

This approach aims at preserving more data constraints in the peripheral to the lakes areas, which are typically removed by the data snooping method.

3.1 Least-squares adjustment (LSA) model

The GRACE and GPS data sets are combined via the least-squares adjustment model

$$\min \left\| \mathbf{l} - \mathbf{A}\mathbf{x} \right\|^2, \quad \mathbf{C}_l \tag{1}$$

where the observation vector $\mathbf{l} = \mathbf{A}\mathbf{x} + \mathbf{v}$ contains the observations $\mathbf{l} = \begin{bmatrix} \mathbf{l}_{GRACE}^T & \mathbf{l}_{GPS}^T \end{bmatrix}^T$ with sub-vectors containing the vertical crustal velocities in the individual data sets. The stochastic model is given by the block-diagonal variance-covariance (VC) matrix

$$\mathbf{C}_l = \begin{bmatrix} (\sigma^2 \mathbf{Q})_{GRACE} & 0 \\ 0 & (\sigma^2 \mathbf{Q})_{GPS} \end{bmatrix} \tag{2}$$

with a scale factor σ^2 and a digonal co-factor matrix $\mathbf{Q}$ of each data set. The calibrated scale factors are estimated by the VCE method.

The coefficient matrix of the parametric model in eq.(1) is

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_0 & \mathbf{A}_{GRACE} \\ 0 & \mathbf{A}_{GPS} \end{bmatrix}, \tag{3}$$

where the sub-matrix $\mathbf{A}_0$ defines a location-dependent offset (a bias, a north-south tilt and an east-west tilt) of the GRACE-derived rates with respect to the GPS/GNSS vertical velocities. The coefficient sub-matrices $\mathbf{A}_{GRACE}$ and $\mathbf{A}_{GPS}$ are formed by means of the function $\Phi(r)$, i.e.,

$$\mathbf{A}_{GRACE,GPS} = [\Phi(r)] \tag{4}$$

where r is the distance between a base function node of a grid that covers the entire region and an observation point. The base functions used are the inverse multiquadrics $\Phi(r) = (r^2 + c^2)^{-1/2}$, which are adapted easily to the data sets by varying the parameter c^2.

3.2 Iterative re-weighting least-squares (IRLS)

According to Hekimoğlu and Berber (2003), an IRLS solution with uncorrelated observations is obtained at the (k+1) iteration as

$$\hat{\mathbf{x}}^{(k+1)} = (\mathbf{A}^T \overline{\mathbf{W}}^{(k)} \mathbf{A})^{-1} \mathbf{A}^T \overline{\mathbf{W}}^{(k)} \mathbf{l},\ \hat{\mathbf{v}}^{(k)} = \mathbf{l} - \mathbf{A}\hat{\mathbf{x}}^{(k)} \tag{5}$$

$$\overline{\mathbf{W}}^{(k)} = \mathbf{C}_l^{-1} \mathbf{W}^{(k)} \tag{6a}$$

$$\mathbf{W}^{(k)} = diag(w_1^{(k)},\ldots,w_i^{(k)},\ldots,w_n^{(k)}) \tag{6b}$$

$$\mathbf{C}_l = diag(\sigma_1^2,\ldots,\sigma_i^2,\ldots,\sigma_n^2)\ , \tag{6c}$$

where $\mathbf{W}^{(k)}$ is the weight matrix at the k^{th} iteration and $\overline{\mathbf{W}}^{(k)}$ is the so-called equivalent weight matrix (Yang, 1994). The weight $w_i^{(k)}$ for observation i is computed by means of the Fair influence function Ψ (Dollinger and Staudte, 1991) as

$$w_i^{(k)} = \Psi\left(\hat{\bar{v}}_i^{(k)}\right) / \hat{\bar{v}}_i^{(k)}\ , \tag{7}$$

where

$$\Psi\left(\hat{\bar{v}}_i^{(k)}\right) = \hat{\bar{v}}_i^{(k)} / \left(1 + \left|\hat{\bar{v}}_i^{(k)}\right| / F\right),\quad F = 1.4. \tag{8}$$

$\hat{\bar{v}}_i^{(k)}$ is the standardized residual

$$\hat{\bar{v}}_i^{(k)} = \hat{v}_i^{(k)} / s \tag{9}$$

where s is the median absolute deviation of the residuals about the median, known as the MAD estimator (Rousseeuw and Croux, 1993):

$$s = med\{|\hat{\mathbf{v}} - med\{\hat{\mathbf{v}}\}|\}. \tag{10}$$

4. ANALYSIS OF RESULTS

The estimated location-dependent offset of the GRACE-derived rates is given in Table 1 for both LSA and IRLS. In both cases, the only significant estimated parameter at the 0.05 level of significance is the bias of 2 mm/yr. This bias is the combined effect of the hydrology correction applied, leakage of signals in the GRACE rates and the effect of the ignored degree one harmonics. By estimating the bias of the GRACE vertical motion rates with respect to the GPS/GNSS velocities, we have implicitly assumed that the line of zero motion is constrained by the GPS/GNSS data.

Table 1. Estimated GRACE data bias (in mm/yr) and tilt (in mm/yr/degree)

Method	Bias	NS tilt	EW tilt
LSA	2.10±0.10	0.12±0.07	0.04±0.04
IRLS	2.06±0.15	0.09±0.07	0.04±0.04

The estimated a-posteriori scale factors of the LSA VC matrices of the GRACE vertical motion rates and GPS/GNSS vertical velocities are 0.1 and 0.6, respectively. The available a-priori VC matrices of both data sets should be downscaled as shown in Table 2. The estimated scale factors of the IRLS VC matrices are 0.01 for GRACE and 0.3 for GPS.

Table 2. Statistics of the a-posteriori error of GRACE vertical rates and GPS/GNSS velocities, in mm/yr

Data set	Min	Max	Mean
	A priori errors		
GRACE	0.6	0.6	0.6
GPS/GNSS	0.5	5.3	2.0
	LSA		
GRACE	0.2	0.2	0.2
GPS/GNSS	0.4	4.2	1.7
	IRLS		
GRACE	0.5	0.8	0.6
GPS/GNSS	0.3	4.1	1.3

The geographical distribution of the GPS/GNSS a-priori (plotted in black) and a-posteriori (red) errors from the LSA and IRLS solutions is depicted in Figures 2 and 3, respectively. A decrease in the GPS/GNSS errors is observed for all points. The large errors are reduced significantly, which is more clearly observed in the IRLS solution.

The vertical motion surface derived by the LSA and IRLS procedures can be compared in the two plots. The line of zero motion is well constrained by the data and does not show any differences in the areas with abundance of points. Subtle differences are noticeable in the northwest peripheral areas. The largest deviations in the vertical motion surface are observed in the west and southwest areas, where few GPS/GNSS stations show a strong correlated signal of -5 mm/yr in a sharp disagreement with GRACE (see Figure 1).

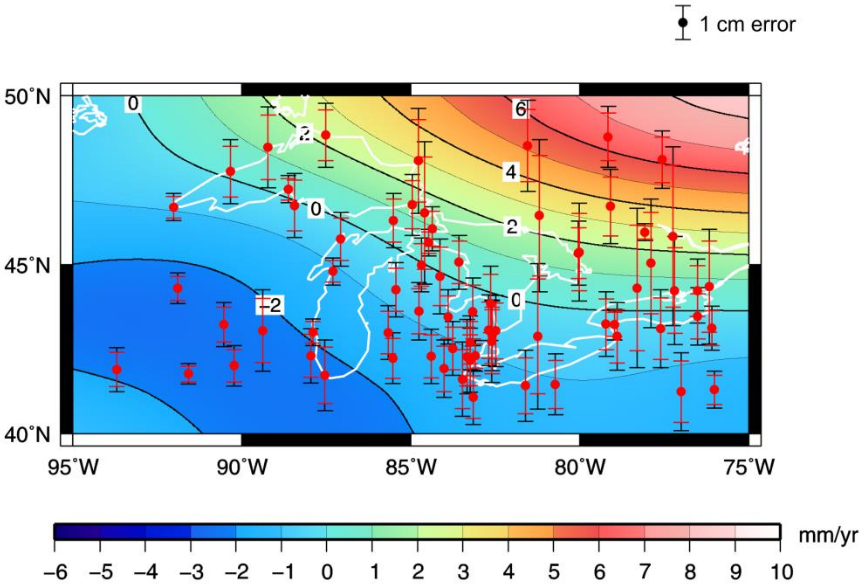

Figure 2. The LSA vertical motion surface with the a-priori (black) and a-posteriori (red) errors of the GPS/GNSS vertical velocities.

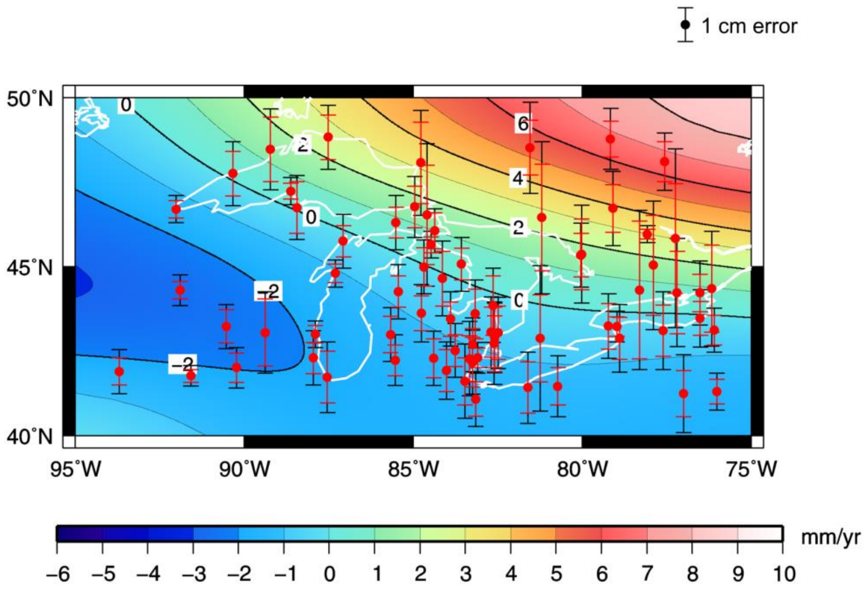

Figure 3. The IRLS vertical motion surface with the a-priori (black) and a-posteriori (red) errors of the GPS/GNSS vertical velocities.

5. CONCLUSIONS

Our earlier investigations have shown that accurate models of vertical crustal motion are indispensible for the maintainance of an accurate datum for heights in Canada, and in the Great Lakes region, in particular, which is characterized by abundance of geodetic data collected in the frame of domestic and international engineering projects. Theoretically, the increasing time span of the GRACE gravity mission would lead to more accurate vertical motion rates that converge to the GPS/GNSS velocities, provided that both geodetic techniques observe the same geodynamic signal. This agreement is hindered by the various error sources and local crustal vertical motion. Based on this assumption, we demonstrated the application of the iterative re-weighting least-squares method for combining heterogeneous vertical motion data with very different error sources and spatial patterns. The main advantage is that if outliers are present in the data, these data points are down-weighted and preserved in the optimal combination, thus allowing for more data in weakly constrained areas. The line of zero motion was well constrained, particularly over the lakes. Detected outliers were not of major concern in our study.

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BIOGRAPHICAL NOTES

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