A Geometric Model for Estimating the Volume and Surface Area of Apples

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SUMMARY

Agricultural and food technology research and science studies frequently require estimates of surface area (SA) and volume (V) of fruits, nuts and vegetables. These estimates are essential for supporting important processes such as storage, sorting, shipping practices, respiration rates, water loss or absorption, use and extent of pesticides and heat transfer.

In this research, low-cost laser scanning and conventional close range photogrammetric methods (for SA) are combined with standard hydro-static techniques (for V) so as to create accurate and scaled 3D models of apples of different sizes and shapes. Based on these models a novel geometric representation for a quick estimation of V and SA of an apple cultivar (i.e. granny smith) is presented. In essence, this geometric model relates to a particular mathematical function referred to as the cardioid.

Accurate statistical figures of V and SA values as obtained via the proposed cardioid representation are determined by comparing the developed model to the points of truth V and SA values obtained from the afore-mentioned 3D models. Results show that for the relatively large apple samples considered the predicted (via the cardioid representation) and the “true” figures for V and SA agree within 2.8% and 3.4% respectively.

The proposed cardioid representation can be defined with simple but precise measurements which comply with international quality standards for size and volume characteristics of apple cultivars in general, that is, apple axial dimensions such as length (L) and diameter (T). Hence, the proposed cardioid model constitutes a practical and effective tool for many applications related to apple measuring, processing and handling.

In particular it may lead to valid applications in orchard management, production and forecasting. In this context, an example describing a field data collection/capture system for apples is carried out in a real environment using a purpose made digital calliper integrating an on-board data-logger and a GPS receiver.

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1. INTRODUCTION

In post-harvest processes the physical properties of agricultural products, such as Volume (V), Surface Area (SA), Density (D) and Weight (W) have wide applications. For instance, one of the critical factors in determining respiratory rate, evaluating colour in separation, heating in heat transfer rate, cooling, and freezing processes is the SA. The other crucial element that plays a role in packaging design and storing agricultural products is the V (Arshad, 2014).

In addition, the requirement of measuring and/or predicting SA and V of fruit items directly or indirectly relate to research studies in all fields of entomology, pathology, physiology, and chemistry. Estimations of SA and V also enter into schemes of areas of fruit damage, sprays, toxic gases and dusts which are applied in the control of pests (Petros et. al, 2014).

Volume of fruit items can be determined via hydrostatic (i.e. water displacement and/or suspension) techniques. Although accurate and reliable, standard hydrostatic techniques are time consuming and impractical under field conditions. Likewise, laser scanning methods for defining V and also SA entail the processing of thousands of sample points (many of them redundant) over the fruit surface, requiring precise surface thinning and matching techniques (Remondino et. al. 2005), not to mention the economics behind the implementation of adequate scanning instrumentation (Ebrahim, 2014).

Image processing methods have also been utilised for determining V and SA of fruit items with various degrees of accuracy results (Blasco et. al. 2012, Mieszkalski et. al. 2017, Sabliov et. al. 2002). In a typical image processing method an intact fruit item is placed on a rotating disk so as to obtain 2D images rectified for distortions and differing by small angles. The contours of the fruit are then extracted from the images and used to reconstruct the 3D dimensional shape of the fruit. The SA of apples can also be determined using the traditional tape method in which strips of narrow masking tape are used to cover each apple surface. The overall surface area is estimated from the length and width of the strips (Clayton 1995, Li 2011).

In view of the accuracy and reliability of a hydrostatic technique referred to as the suspension method and current laser scanning processes, these were considered here as a way to establish a point of truth for comparison and thereby prove statistically the validity and/or the accuracy attainable with the proposed cardioid model. The proposed model offers a much less sophisticated and more practical approach and it is based upon assessing and/or measuring apple axial dimensions, namely, length (L), and diameter (T) of a given apple, see Figure 1(a) and 1(b). These dimensions can be determined by way of simplistic although precise measuring equipment such as callipers as shown in Figure 1(a), or can be derived as illustrated in Figure 1(b) from measurements taken with a Cranston gauge whereby the diameter of the apple can easily be read from the gauge. The measurements of these dimensions are along the lines of international quality standards which state that the size of apples needs to be measured at their equatorial part (Arshad, 2014). This is by some means reflected in the T and L dimensions in Figure 1.

(a)

(b)

(c)

(d)

Figure 1 - Using of a calliper (a) and/or a Cranston gauge (b) to determine the two principal dimensions (L: Length, T: Thickness) of an apple.

2. THE CARDIOID MODEL

From a deterministic viewpoint, and making an allowance for the geometry depicted in Figure 1, SA and V of apples may be estimated upon the assumption that they tolerate a similarity to axis-symmetric geometric shapes from which these SA and V can be mathematically approximated. Two such traditional models are spheres and ellipsoids (Sabliov et. al., 2002, Galbreath 1976). However, it was found that the sphere and ellipsoid models underestimated the actual SA of apples by 15% and 18% respectively (Clayton 1995). Feasibly, a better geometric shape would be that of considering an apple comparable in shape to a cardioid of revolution. Figure 2 shows how the section of an apple (Granny Smith cultivar) adheres to a cardioid representation.

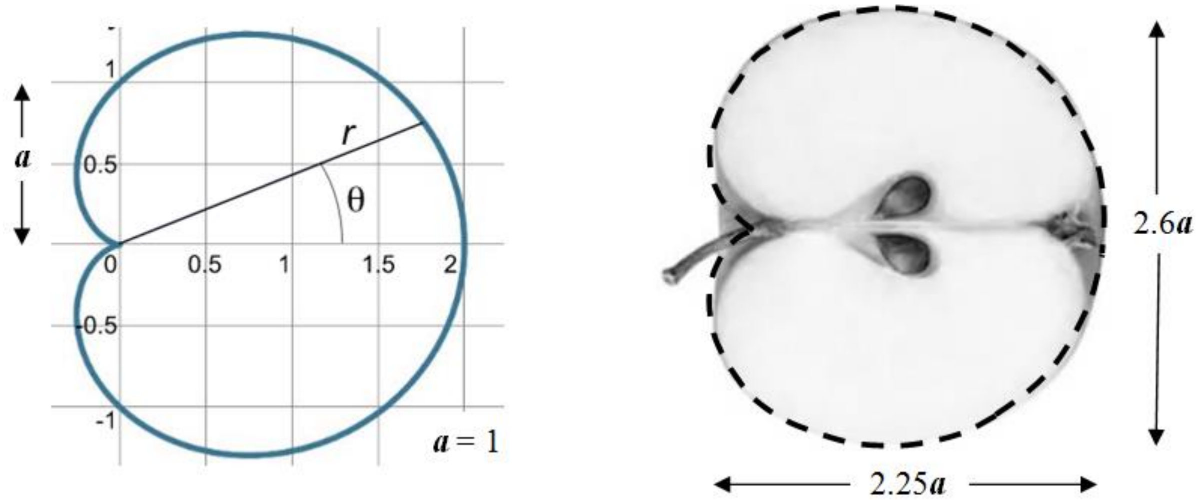

Figure 2 - Cardioid representation for ***a*** = 1 (left) and a section of an apple and its dimensions in terms of the same parameter ***a***.

A cardioid shape can be created by following the path of a point on a circle as the circle revolves around another preset circle, with both of the circles having the same radius. When it comes to the equations of cardioids, polar form is usually used for simplicity. The polar form of an equation involves polar coordinates instead of rectangular (meaning x and y) coordinates. As per Figure 2 polar coordinates are points (r, θ) that are plotted on a polar coordinate system in which r is the length of the line segment connecting the point to the origin and θ is the angle that is created between the polar axis and the line segment from the point to the origin (Akopyan 2015).

Therefore, the polar form of an equation has variables r and θ, and is satisfied by the points (r, θ) that make the equation true. There are two possibilities: a horizontal cardioid and a vertical cardioid. If the radius of the circle that creates the cardioid is ***a***, then the equation of a horizontal cardioid is as per equation (1) whereas the case of a vertical cardioid is as per equation 2.

$$r = \boldsymbol{a} \pm \boldsymbol{a}cos\theta \qquad (1)$$
$$r = \boldsymbol{a} \pm \boldsymbol{a}sin\theta \qquad (2)$$

the equation of a vertical cardioid was used in this study, thus taking the form of $r = \boldsymbol{a}\text{-}\boldsymbol{a}\sin\theta$ as per equation (2). Consequently, the estimation of V and SA of this study is based on a mathematical model that in essence relates to the computation of V and SA of a cardioid of revolution. A three-dimensional solid of an apple model is so obtained by rotating the cardioid around the stem-calyx axis. From the theory of calculus (Ayres 2008) it can be demonstrated that the SA of revolution of the cardioid shown in Figure 2 can be expressed as per equation (3) whereas V can be given by the expression in Equation (4)

$$SA = \frac{32*\pi*\boldsymbol{a}^2}{5} \quad (3) \qquad\qquad V = \frac{8*\pi*\boldsymbol{a}^3}{3} \quad (4)$$

From Figure 2, the value of $\boldsymbol{a}$ is determined from the apple measurements by multiplying the diameter perpendicular to the core (T) by 0.385 (i.e. 1/2.6= 0.385) and/or the diameter parallel to the core (L) by 0.444 (i.e. 1/2.25=0.444).

This axis-symmetric representation greatly reduces the amount of modelling work and calculation, yet it may still yield improved estimates of an actual apple V and SA as compared to sphere and ellipsoid representations. However, since apples may not be generally classed as axis-symmetric items, and in order to further increase the accuracy of the results, improved estimates for V and SA can be determined by way of using and averaging multiple values of V and SA as obtained by measuring L and/or T of a given apple at different locations.

Averaging the results of V and SA using multiple measurements of L and T relates to the fact that noise (i.e. abnormally shaped or relatively deformed apples) and measurement inaccuracy (i.e. improper measurements of L and T) are random, and therefore, by the Central Limit Theorem, the error will have a normal (Gaussian) distribution. That is, by averaging multiple measurements, a Gaussian distribution can be established. Hence, an actual average that is statistically close to the actual value can be achieved. In other words, the more measurements of L and T are averaged, the more accurately the actual values of V and SA will be.

3. POINTS OF TRUTH

The true values for V were computed by way of measuring the volume of small objects based on the Archimedes principle referred to as the suspension method. It involves suspending an object (i.e. an apple) in a water-filled container placed on an electronic scale. The principle is based on the fact that said object can simply be suspended in water and the change in weight translated directly into its volume.

This method was selected as it is a technique that has proven to be more accurate and precise than the traditional water displacement method. The reader is referred to Hughes (2005) for a thorough explanation of this method of volume determination, including its advantages when compared to the standard water displacement method.

On the other hand, the true value for determining the SA was based on a laser scanning process supported by close range photogrammetric techniques for small objects (Galantucci et. al. 2015). Close range photogrammetry was mainly used to corroborates the accuracy of the measurements as obtained from the scanning process and only involved a small number of apples. This technique entailed many time consuming steps (i.e. scaling, camera calibration, image rectifications and matching), plus requiring tarhet poins distributed over the fruit surface.

On the other hand, laser scannning produced more stable and consistent results. The scanner available to the authors is shown in Figure 3 and is referred to as Matter and Form 3D Scanner (https://matterandform.net/scanner), with a scanning accuracy within 0.1 mm (as claimed by the manufacturer). SA for each apple was time consuming and it was computed based on 2500 randomly scanned xyz points (appximately 8-10 points per cm^2) which were used to generate a triangulated irregular network or TIN model. Note that each apple had to be scanned at 3 different positions to account for the stem and calyx areas.

The surfaces obtained by the three scans were thereafter aligned to achieve complete geometry, that is, a complete 3D model for each apple. Alignment was generated by exporting the TIN model to Meshlab (http://www.meshlab.net/) so as to determine the final SA. Meshlab is an open source system software for processing and editing 3D triangular meshes. Meshlab was also used to determine V figures for each apple. Upon comparing the V results obtained by the above-mentioned suspension method and the V obtained by the scanning process it was found that overall they only differ by 0.26%. This gave the authors assurance regarding the reliability of the adopted point of truth values for V and to a certain extent the values of SA.

Figure 3 - The “Matter and Form” laser scanner used to determine the SA of a selected apple. The results from the scan were also used as an alternative method of computing the V.

The scanning process was accomplished in approximately 7-8 minutes per individual apple provided the scanner is set on the QuickScan mode. However, the time needed for the computation of SA via Meshlab was in the vicinity of 10 minutes per individual pear. This involved the so-called thinning or elimination of duplicate and/or redundant overlapping points of the dense cloud of points produced by the scanner.

4. MATERIALS AND METHODS

Fifty (50) consumer grade apples from the Granny Smith cultivar were selected for the tests. All apples originated from orchards in the *Stanthorpe* area. Stanthorpe is a town situated in south east Queensland, Australia. Selection was also based on apples showing relative irregular shapes, with the exception of very abnormally or markedly asymmetrical fruit, but otherwise random samples were used. Fruit weight (W) of the samples ranged between 100 gr. to 320 gr. (see three typical samples in Figure 4). They were measured to the nearest 0.1 gr. and fruit V determined by the suspension method.

In line with the principle and dimensions in Figure 1, measurements of each apple consisted of four perpendicular transverse measurements and two longitudinal measurements between the upper and lower extremities of the fruit. Measurements were taken with a digital calliper to the nearest 0.1 mm.

Actual apple SA was estimated by way of scanning each apple as per the scanning technique mentioned in the previous section. Accuracy of the technique was further verified by scanning a near perfect sphere of known radius/diameter (a solid fiberglass ball) in the same way. Estimation of SA for the sphere by the scanning method closely approximated (0.35% difference) the value obtained by mathematical calculation based on the sphere diameter.

Predicted values of V and SA using the proposed cardioid model were obtained for each apple by using the average of *a* from four measurements of L and T taken on the apple. Statistical accuracy figures (RMSE) were determined between the point of truth V and SA and the predicted values as obtained from the proposed cardioid model.

Figure 4 - Three samples of the 50 apples selected (Granny Smith cultivar) for the tests. All apples presented some shape imperfections and range in W between 100 gr. to 320 gr.

Table 1 illustrates the RMSE of the differences between the point of truth values of V and SA and the predicted values. The differences in terms of the overall percentage are also included. Table 1 also indicate the maximum and minimum deviation from the "point of truth" values.

	RMSE	Overall Diff. %	Max. Diff.	Min. Diff.
Surface Area	+/- 2.23 cm^2	3.4	4.75 cm^2	- 3.33 cm^2

Volume	+/- 3.15 cm^3	2.9	3.45 cm^3	- 4.08 cm^3

Table 1 - RMSE of the differences between predicted values of SA and V using the cardioid model and those determined by the point of truth method. The overall difference in percentage and maximum and minimum deviation from the point of truth values of SA and V are also shown.

5. CONCLUSIONS AND DISCUSSION

This study was conducted in March 2019 in a commercial apple orchard located in Stanthorpe, Queensland, Australia. Data were collected from a 1 Ha (approx.) block of Granny Smith apple trees planted in 2012 at the density of approximately 360 trees per ha.

The maximum equatorial diameter of 21 apples on 14 randomly selected trees among the rows for a total of 294 apples was recorded following the idealised diagram illustrated in Figure 5 by Manfrini et. al. 2015.

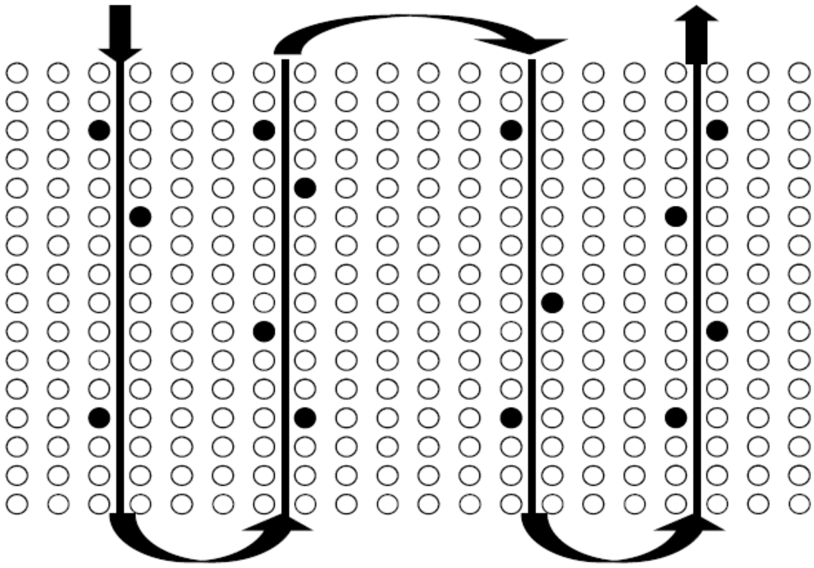

Figure 5 - A random tree selection strategy for data collection (Manfrini et. al. 2015). For each selected tree (black dot) 21 maximum diameter measurements were carried out.

Data were recorded using a digital calliper fitted with an on-board data-logger. A GPS enabled digital camera (Nikon CoolPix AW130) recorded the tree locations and pictorial information. This measuring system allowed for a single person to gather data for all 294 measurements in about 1.5 hour each time. It is envisaged that the measurement as per the above system could be applied during the fruit phase of growth. The data captured this way can be processed and statistical results released to the grower so as to evaluate the progress of the crop, and for substantiation that the applied management techniques are yielding the expected productivity. This statistical forecast could also be used to evaluate a relationship between predicted and the real breakdown of the total yield into size and shape groups, which would also allow for the release of an accurate educated guess of overall orchard yield for packing purposes.

6. CONCLUSIONS AND DISCUSSION

The goal of this research was to develop a model that allows a mathematical description of the geometry of apples with only a few parameters. The cardioid modelling approach described here provides a simple and practical tool for characterizing important apple physical dimensions such as V and SA.

The study findings confirmed the capability and accuracy superiority of the cardioid model in predicting V and SA of apple fruit as compared to other mathematical models based on sphere and/or ellipsoid geometries. The proposed model further reduces complicated mathematical calculations, making it easy to program said calculations using conventional office software such as Microsoft Excel.

Clearly, methods of measuring V and SA as related to the water suspension, laser scanning techniques and close-range photogrammetry would yield more accurate results but these methods are time consuming, they require particular (and expensive) measuring equipment and are impractical for field measurements (i.e. assessing apples still attached to trees).

The following conclusions can be drawn from this study:

- The adopted cardioid model can define the representative geometry of processing apples in terms of minimal dimensional parameters and shape coefficients.
- The proposed three-dimensional model of apple was validated satisfactorily in terms of precision and accuracy with experimentally measured apple geometric data.
- The established cardioid apple model has a good potential for application in apple-related computer aided designs and simulations and facilitates ideas for further geometric modelling of agricultural products.

Further studies are needed to determine whether the cardioid model or transformations of said model (Cervantes et. al. 2016) may be suitable or match other apple cultivars different than the one examined here (i.e. Granny Smith cultivar). For instance, from the model generalisation of apple shapes (Ziaratban et. al. 2017) given below in Figure 6 it can be inferred that the elongated shapes A and B are more like Delicious apples, while the rounder models C and D bear a resemblance to 'Golden Delicious', 'McIntosh' and Granny Smith apples. Model A may be considered a variant of model B (or model C a variant of model D) only in the way the sections are configured.

Hence, departing from these curves, it may be possible to design new geometric models adapted or associated with other apple cultivars. Modelling these shapes using different cardioid transformations may allow for comparisons, general identifications and/or determining differences between the physical attributes related to a variety of apple cultivars

(Reid 1976). It should be noted that the four apple models presented in Figure 6 have the same V of revolution but different SA of revolution. Hence, having the same V certainly does not mean that the SA is the same. Finally, all apples included in the above tests were consumed and none of them were disposed or destroyed.

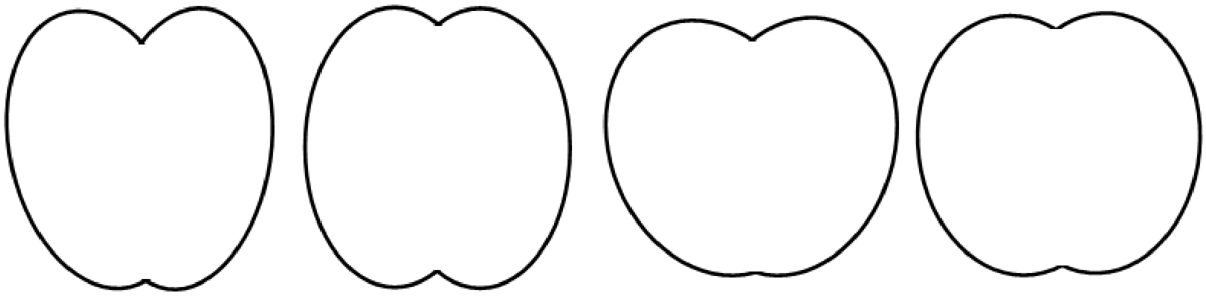

Figure 6 - Four apple models, all of which have the same volume of 210 cm^3, matching to a medium apple size, based on an estimated mass density of 0.815 g./cm^3

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